A logical framework†

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Resumen

El artículo presenta un marco de distinciones para la filosofía de la lógica en la que las interrelaciones entre algunas nociones lógicas centrales, como la de declaración, juicio (el acto), juicio, (el resultado de juzgar), proposición (contenido), consecuencia e inferencia, se detallan.
PALABRAS CLAVE: Declaración, juicio, proposición, consecuencia, inferencia.

Abstract

The paper presents a framework of distinctions for the philosophy of logic in which the interrelations between some central logical notions, such as statement, judgement (-act), judgement (made), proposition (al content), consequence, and inference are spelled out.
KEY WORDS: Statement, judgement, proposition, consequence, inference.

1. Hilary Putnam, and, following him, George Boolos, have, on different occasions, taken exception to Quine's dictum that

> "*Logic is an old subject, and since 1879 it has been a great one*",

with which he opened the first editions of his *Methods of Logic*.[1] In their opinion, Quine's implicit preference for Frege's *Begriffsschrift* does an injustice to Boole (Boolos and Putnam) and the Booleans, of whom Peirce in particular (Putnam). Ten years ago, in an inaugural lecture at Leyden, also I argued that Quine presented too narrow a view of logic, and, that as far as the nineteenth century was concerned, the crucial date in the development of logical doctrine is not 1879 (nor 1847, I would add today, disagreeing with Boolos' stimulating paper), but 1837, the year in which Bernard Bolzano published his treatment of logic in four hefty volumes.[2]

Why does this Bohemian priest deserve pride of place, over and above such luminaries as Boole, Peirce and Frege? For more than two thousand years, logic had been concerned with how to effect valid acts of inference from judgements known to other

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[1] Putnam (1982), Boolos (1995), and Quine (1952).

[2] *Oordeel en Gevolgtrekking. Bedreigde Species?,* an inaugural lecture delivered in Leyden University, September 9, 1988, and published in pamphlet form by that university.

judgements that become known through the inference in question. Basically, these judgements take the subject/copula/predicate form [S is P]. Bolzano now has the courage to break with this traditional pattern and uses instead the unary form

(1)   A is true,

where A is a *Satz an sich*, or a *Gedanke*, in the later alternative terminology of Frege. The latter term was translated into English as *proposition* by Bertrand Russell, with an unusually confusing ambiguity as a result: prior to 1900 a "proposition" stood for a judgement (made), whereas later it came to stand for the propositional content of such a judgement. For Bolzano, logic was very much concerned with knowledge; his critical examination and exposition of *logic* is called *Wissenschaftslehre* [An approximate translation might be *The Theory of (Scientific) Knowledge*.]. Just as his main target Kant, he holds that a correct (*richtig*) judgement is a piece of knowledge (*ein Erkenntnis*).[3] To my mind, he is perfectly right in doing so. After the "linguistic turn", in place of judgements, one can consider instead the proper form, and relevant properties, of their linguistic counterparts, namely assertions. An assertion is effected by means of the assertoric utterance of a declarative sentence. This explanation must be supplemented with a criterion of assertoric force, on pain of a vicious circularity. Such a criterion is provided by means of the question:

(2)   How do you know? What are your grounds?

which is legitimate as a response to an assertion. In case the utterance was assertoric, the speaker is obliged to answer, and if he cannot do so the assertion was *blind*. The assertion issued by an act of assertion, but for what is stated, also contains an illocutionary claim to knowledge. Thus, I am able to make public my knowledge that snow is white through the assertoric utterance of the declarative

(3)   'Snow is white'.[4]

The explicit form of the assertion thus made would then be:

(4)   I know that snow is white.

A sole utterance of the nominalization

(5)   that snow is white,

which expresses the propositional content, on the other hand, will not so suffice. A phrase sufficient for the making of assertions is reached by appending 'is true' to the nominalization in question. An assertoric utterance of the declarative

(6)   that snow is white is true

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[3] (1837, §34).

[4] The example 'snow is white' is taken from Boole (1854, p. 52).

does suffice to effect an assertion with (4) as the assertion made. The fully explicit form, with indication of knowledge, and truth of content, accordingly becomes:

(7)   I know that that snow is white is true.[5]

This grammatically necessary, but hardly idiomatic, iteration of *that* can be avoided here through the transformation:

(8)   that S is true = it is true that S,

which yields

(9)   I know that it is true that snow is white

as the explicit form of the assertion made through an utterance of (3).
I do not mean to imply that this was the route that Bolzano actually took to his novel form of judgement: it was not. I have used various linguistic considerations concerning the form of assertions when viewed as reports of knowledge, whereas Bolzano insisted that his *Sätze an sich* were completely independent of all matters linguistic and cognitive. Be that as it may; the argument given provides a rationale for why correct judgements made are pieces of knowledge, and why the proper form of judgement is "truth ascribed to propositional content".
There remains the problem of choosing an appropriate terminology for entities in the expanded declarative form (6). Frege held that declarative sentences expressed *propositions*, that is, for example, the declarative *snow is white* expresses the proposition that snow is white. I prefer not to join Frege in this. Wittgenstein used the terminology *Satz* and *Satzradikal*. The latter, clearly, is the proposition(al content), but the former has to do double duty for declaratives and what they express. For my purposes the best choice here might well be *statement*.[6] Sentence, statement and proposition then serve in different logical roles:

(10)   a declarative sentence expresses
          a statement with a proposition as content.

Thus,
(11)   the declarative 'snow is white' expresses that snow is white.

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[5] If the iterated *that* offends, a use of *it is true that … in* place of *… is true* provides relief.

[6] *Enunciation* and *declaration* are other alternatives. The latter has a certain high-sounding ring to it, but might otherwise have served very well. My discussion could then have been pithily summarised:
            The assertoric utterance of a declarative makes an assertion that claims knowledge of the declaration expressed (declared?) by the declarative in question.
My preference for statement is, i. a., based on the fact that *statement* is the English term which is applied to reports by witnesses. This is followed also in German: *Aussage, Zeugnis*, and Swedish: *utsaga*, whereas Dutch: *Declaratie, verklaring* uses *declaration* instead. Also MacColl (1880, p. 53) links his use to the legal one, for which reference I am indebted to Shaid Rahman. This advantage has to be weighed against the drawback that since Cook Wilson — *Statement and Inference* — the term has been in constant Oxford use, where it has served in many roles, among which those of, propositional content (with indexicality taken into account; perhaps, after E. J. Lemmon (1966), the most common current use), declarative sentence, and state of affairs the act of saying, and what is said, the act of asserting, and the assertion made.

When one steps over to the expanded statement-form an iteration of *that* occurs,

> (12)   'snow is white' expresses that that snow is white is true,

which can be removed using the transformation (8)

> (13)   the declarative 'snow is white' expresses that
>   it is true that snow is white.

Consider now an act of assertion made through an assertoric utterance of the declarative sentence 'Snow is white'. With respect to the assertion made the discussion above can be summarised in the following table:

Assertion made (explicit form)	I know that it is true that snow is white.
(Illocutionary) Knowledge-claim	I know that
Statement that is asserted	that snow is white is true =
	it is true that snow is white
Propositional content	that snow is white

Note here also that judgement is often used instead of *statement* and *assertion*, both with respect to the act and object. Thus propositions have truth-conditions, whereas statements(judgements) have assertion-conditions.

The implication A implies B, in symbols A⊃B, between two propositions A and B, is another proposition, which accordingly is a candidate for truth. Classically A⊃B is true when A is false or B is true, whereas its constructive truth consists in the existence of a suitable proof-object. It should be stressed that 'implies' can only join propositions, but not statements: the proposition that grass is green implies that snow is white is fine from a grammatical point of view, whereas an attempted connection between statements yields the nonsensical 'grass is green implies snow is white' which, as Quine noted, contains too many verbs.[7] Propositions can also be joined into a relation of consequence, which yields a generalisation of propositions:

> (14)   the consequence from A to B,

in (Gentzen-like) symbols A⇒B.[8]

The consequence, or sequent, A⇒B holds precisely when the corresponding implication A⊃B is true (also constructively). Much to his credit, Bolzano considered also this notion of consequence — he called it *Ableitbarkeit* — whereas today one is only interested in the *logical* holding of the consequence. (A consequence holds logically when the corresponding implication is a logical truth, that is, is true come what may, independently of what is the case.) It should be clear that the inference

$$\frac{A\Rightarrow B \text{ holds} \qquad\qquad A \text{ is true}}{B \text{ is true}}$$

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[7] Quine (1940).

[8] Consequences between statements will, not work for the Quinean reasons. Cf. the preceding footnote.

is perfectly valid as it stands; one does not need the logical holding or the logical truth in the premises in order to be allowed to conclude that B is true. (Similarly, we do not need the *logical* truth of A⊃B in order to draw the conclusion B is true from the premise that A is true.)

Just as we can combine propositions both into implications, which are propositions, and consequences, which are not, statements can be combined into conditionals, which are statements, and inferences, which are not. For example, a *conditional statement* results from applying, not categorical, but hypothetical truth

(15)   … is true, provided that A is true,

to a proposition:

(16)   B is true, provided that A is true.

The *proviso* can also be expressed in other ways: on condition that, under the hypothesis that, assumed that, etc., will all serve equally well here. Conditional statements can be obtained also in other ways, for example by joining statements by means of *If-then*:

(17)   If A is true, then B is true.

The assertion-conditions for the three statements

A⊃B is true,
A⇒B holds,
B is true, provided that A is true, or, in another formulation,
If A is true, then B is true

are different (we have not got the same statement three times over), but if one is entitled to assert any one of these, the requirements for asserting the other ones can also be met. Finally, an inference is, in the first instance, a mediate act of judgement, that is, (taking the linguistic turn) an act of asserting a statement on the basis of other statements being already asserted (known). So the general form of an inference I is:

$$\frac{J_1 \ldots J_k}{J.}$$

The inference I is valid if one is entitled to assert J when one knows (has asserted) J_1, … J_k. Accordingly, in order to have the right to draw the inference I must possess a chain of immediately evident axioms and inferences that link premises to conclusion.[9] After Bolzano it has been common to conflate the validity of the inference I'

$$A_1 \text{ is true},\ldots ,A_k \text{ is true}$$

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[9] This notion of validity is age-old. Compare Quine and Ullian (1970, p. 22) for a recent formulation: 'When a…truth is too complicated to be appreciated out of hand , it can be proved from self-evident truths by a series of steps each of which is itself self-evident — in a word it can be deduced from them.'

$$C \text{ is true.}$$

with the logical holding of the consequence $A_1, \ldots, A_k \Rightarrow C$. That is, one reduces the validity of the inference to the logical holding of a relation of consequence between the propositional contents of statements that serve as premises and conclusion, respectively, of the inference in question. Bolzano also reduced the correctness of the statement that the rose is red is true to the rose's *really* being red. In both cases, the reduction gives rise to what Brentano called *blind judgements*: a judgement can be correct, by fluke, even though the judge has no grounds, and similarly for blind inference.

Bolzano's other notion of consequence — that of *Abfolge* — is less clear, but can perhaps be understood in the following way. Consider the inference

(18) S_1. Therefore: S_2.

In expanded form it becomes:

(19)   That S_1 is true. Therefore: that S_2 is true.

When this inference is drawn and made public through an utterance of (18), we have assertions of (i) the premise that S_1 is true, (ii) of the conclusion that S_2 is true, and (ii) of the inferential link between them. Instead of considering the validity of the inference, Bolzano's *Abfolge* involves a *propositional operator* …entails…such that

(20)   The proposition that S_1 entails that S_2 is true =
            the inference (18) is valid, and
            the premise <u>that S_1 is true</u> is correct.

Bibliography

Bolzano, Bernard: *Wissenschaftslehre*, J. Seidel, Sulzbach, 1837.

Boole, George: *The Laws of Thought* (1854), Macmillan, London (Dover, N.Y.) 1958.

Boolos, George: *Reading Putnam* in: Bob Hale and Peter Clarke (eds.), Blackwell, Oxford, 1994, pp. 31-48.

Putnam, Hilary: "Peirce the logician" in *Historia Mathematica* **9**, 1982, pp. 290-301.

Quine, Willard Van Orman: *Mathematical Logic*, Norton, New York, 1940.

Quine, Willard Van Orman: *Methods of Logic*, Holt and Co., New York, 1950.

Quine, W. V., and J. S. Ullian: *The Web of Belief*, Random House, New York, 1970.