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Images and Logic of the Light Cone: Tracking Robb's Postulational Turn in Physical Geometry*

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Abstract

Previous discussions of Robb's work on space and time have offered a philosophical focus on causal interpretations of relativity theory or a historical focus on his use of non-Euclidean geometry, or else ignored altogether in discussions of relativity at Cambridge. In this paper I focus on how Robb's work made contact with those same foundational developments in mathematics and with their applications. This contact with applications of new mathematical logic at Göttingen and Cambridge explains the transition from his electron research to his treatment of relativity in 1911 and finally to the axiomatic presentation in 1914 in terms of postulates. At the heart of Robb's physical optics was the model of the light cone. The model underwent a transition from a working mechanical model in the Maxwellian Cambridge sense of a pedagogical and research tool to the semantic model, in the logical, model-theoretic sense. Robb tracked this transition from the 19th- to the 20th-century conception with the earliest use of the term 'model' in the new sense. I place his cone models in a genealogy of similar models and use their evolution to track how Robb's physical researches were informed by his interest in geometry, logic and the foundations of mathematics.

Keywords: Robb, axiomatics, postulates, postulationism, light cone, relativity theory, geometry, foundations of mathematics, space-time, model, logical model, Russell, Hilbert, Veblen, Huntington, Peano, Minkowski, Cambridge, Göttingen.

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Las imágenes y la lógica del cono de luz: rastreando el giro postulacional de Robb en la física geométrica

Resumen

Las discusiones anteriores de la obra de Robb acerca del espacio y el tiempo han ofrecido un enfoque filosófico de las interpretaciones de la teoría de la relatividad o un enfoque histórico de su empleo de la geometría no-euclidiana, o han ignorado enteramente las discusiones de la relatividad en Cambridge. En este artículo centro mi atención en la forma cómo la obra de Robb tomó contacto con esos mismos desarrollos fundacionales en la matemática y con sus aplicaciones. El contacto con las aplicaciones de la nueva lógica matemática en Göttingen y en Cambridge explica la transición de las investigaciones de Robb sobre los electrones a su tratamiento de la relatividad en 1911 y finalmente a su presentación axiomática de 1914. En el corazón de la óptica física de Robb estaba el modelo del cono de luz. Este modelo pasó de ser un modelo mecánico operante en el sentido cantabrigense maxwelliano de herramienta didáctica y heurística a ser un modelo semántico en el sentido lógico de la teoría de modelos. Robb marcó esta transición de la concepción del siglo XIX a la del siglo XX con el uso más temprano del término “modelo” en el nuevo sentido. Sitúo sus modelos de conos en una genealogía de modelos similares y uso su evolución para seguir la pista de cómo las investigaciones físicas de Robb dependían de su interés en la geometría, la lógica y los fundamentos de las matemáticas.

Palabras clave: Robb, axiomática, postulados, postulacionismo, cono de luz, teoría de la relatividad, geometría, fundamentos de las matemáticas, espacio-tiempo, modelo, modelo lógico, Russell, Hilbert, Veblen, Huntington, Peano, Minkowski, Cambridge, Göttingen.

Introduction

During the pre-war period, physicists at Cambridge were working at the forefront of electron and radiation research. While it engaged results in the Continent, especially in the Netherlands and Germany, it was rooted in local tradition of theoretical and experimental research in electromagnetism. Two Cambridge researchers, Norman Campbell and Alfred Arthur Robb, merit special attention for their additional interest in relativity theory and logical dimensions of scientific thinking. Europe-wide concerns about the relation between matter and energy or the optics of moving bodies made relativity theory relevant to the study of radioactivity just as they had motivated Einstein in the first place. More intriguing is their logical point of view and its application to physics. Elsewhere I address how new developments in the foundations of mathematics, especially the logical study of axiomatics, guided the American appropriation of relativity theory and this, in turn, influenced German formulations (Cat, 2016). In this paper I focus on how Robb's work made contact with those same foundational developments in mathematics and with their applications. This contact with applications of new mathematical logic at Göttingen and Cambridge explains the transition from his electron research to his treatment of relativity in 1911 and finally to the axiomatic presentation in 1914 in terms of postulates. Previous discussions of Robb's work on space and time have offered a philosophical focus on causal interpretations of relativity theory or a historical focus on his use of non-Euclidean geometry, either in the context of a new tradition of German geometrical physics in Minkowski's footsteps or in the context of a Cambridge tradition of research in geometry –or else ignored altogether in discussions of relativity at Cambridge.[1] Instead, I emphasize the synthesis of logical and physical aspects of his interest in axiomatics by drawing attention to two intersecting contexts of his pre-war application of axiomatics in physical geometry: his evolving engagement of mathematical logic and the continuity with his early electron research.

Geometry and optics paved Robb's path to relativity. In this path, Robb's particular conception of geometry was key, it combined the two dimensions of a physical and formal theory; the first providing physical grounds and interpretation and the second, structure. The duality was rooted in his dual interests and education in geometry and in Cavendish physics. In particular, it relied on the dual character of optical theory as a geometrical theory and a physical theory situated at the core of Cavendish research on electromagnetic radiation. At the heart of Robb's physical optics was the model of the light cone. The model underwent a transition from a working mechanical model in the Maxwellian Cambridge sense of a pedagogical and research tool to the semantic model, in the logical, model-theoretic sense. Robb tracked this

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[1] For a brief discussion of the causal philosophical analysis see Torretti 1983/1996, 123; for the historical focus on non-Euclidean geometry see Walter (1999: 17-8), and Barrow-Green and Gray (2006: 337-9); Barrow-Green and Gray situate Robb at the intersection of geometry and relativity research at Cambridge alongside E. C. Cunningham and A. Eddington, noting the axiomatic approach, as does Darrigol in Darrigol (2014: 139-42); Warwick leaves Robb out of his analysis in Warwick (2003) of the appropriation of relativity theory by Cambridge physicists.

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transition from the 19th- to the 20th-century conception with the earliest use of the term 'model' in the new sense. The evolving meaning of cone models illustrates and tracks Robb's academic itinerary, from Cambridge to Göttingen and back. It tracks also the parallel evolution of geometric theory through the late 19th-century history of European mathematics, from the standard of material axiomatics to the new logical approach to modern axiomatics introduced by Hilbert and the Italian school, developed differently by American mathematicians and by Russell at Cambridge. Finally, the evolution of cone models tracks how Robb's physical researches were informed by his interest in geometry, logic and the foundations of mathematics.

1. Relativity at Cambridge before the war, from an electromagnetic point of view

To appreciate the specificity of Robb's logical turn and its application to his interest in geometry they should be set against the features of the local physics tradition. Those shared features led others to a predominant approach engaging relativity theory that I will describe briefly; yet Robb's approach was markedly different. If fact, in a comprehensive study of this engagement at Cambridge the emphasis on the role of the local tradition includes the added role of an explicit rejection of Einstein's and other German physicists' "axiomatic" presentations, that is, based on principles introduced without further justification (Warwick, 2003).[2] But this was precisely also Robb's approach to investigating space and time, which he pursued more explicitly and rigorously. His axiomatic approach was more in line with the new approaches in geometry and other branches of mathematics in Germany, America and Cambridge.

The culture of mathematical physics at Cambridge before the First World War had been handed down through teaching and research since Maxwell's tenure through the 1870s, and developed further especially by researchers trained mainly at the Cavendish laboratory. Maxwell had introduced a project of mathematical physics based on the application of differential equations to represent the energy states of the electromagnetic ether. The propagation of waves in the elastic ether provided the basis for Maxwell's synthesis of electromagnetism and optics. In its wake, British electromagnetic research was based on mechanical models of the ether (Hunt, 1991).

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[2] Despite its emphasis on the diversity of approaches at Cambridge, Warwick's account identifies some diversity only within the application of the local standards leading to the predominant approach and leaves out Robb's.

In the 1890s Joseph Larmor and J.J. Thomson added the existence of microscopic electrons to the fundamental understanding of matter and electromagnetic radiation. Taking a step further, they declared mechanical properties of matter of to be derivative from the more fundamental electromagnetic properties of electrons and the ether. Larmor insisted on the even more fundamental role of the principles of energy and least action governing the ether. The electrons that constituted matter were also properties of the ether, movable singularities. The new theory offered a unification of matter theory, optics and electromagnetism (Larmor, 1900).

In several experiments in the 1880s Albert Michelson and Edward Morley attempted to detect the influence on electromagnetic effects, including the propagation of light, of motion through the ether. Larmor's electron theory could also derive the hypothesis of the contraction of electronic matter moving through the ether, which George FitzGerald in 1889 and independently H.A. Lorentz in 1892 had introduced to explain Michelson and Morley's null result. The space-time transformations that implied the contraction of matter also left invariant Maxwell's equations of electromagnetism. For Larmor the experimental results gave support to his fundamental physical theory and the invariance placed Maxwell's theory and the speed of light at its center.

Pre-war reactions to Einstein's theory at Cambridge included correspondence from G. F. C. Searle and work by Ebenezer Cunningham, Harry Bateman, G.A. Schott, H.R. Hassé, S.B. McLaren and J.W. Nicholson. In the late 1908, the German electron physicist Alfred Bucherer got Einstein to mail Searle a copy of a review article on the theory of relativity.[3] Searle wrote back acknowledging apologetically the lack of understanding around him of the principle of relativity. By then Cunningham was engaged in electron research and followed Larmor in justifying the Lorentz transformations on the grounds that they preserved Maxwell's equations and that the implied contraction of moving matter was predicted by the theory of the ether.

For Cunningham and many others, Einstein's theory, known as the Lorentz-Einstein theory, was an electron theory with a new presentation. The principle of relativity, the invariance of the laws of physics under the transformations for uniform motions, was not fundamental; nor was the accompanying principle of the constancy of the speed of light, which for him followed from the first. Einstein's contribution was a mathematical treatment that emphasized the geometrical symmetries of space-time in the axiomatic

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[3] Warwick discusses the episode in Warwick (2003: ch. 8). I borrow a selectively from the same chapter for the brief sketch of the Cambridge reaction.

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spirit but that were nevertheless derivative from the electromagnetic nature of the ether (a view also shared by Schott, Hassé and McLaren). With Bateman, Cunningham next sought a "new theorem of relativity" that generalized the "geometrical" transformations preserving Maxwell's equations to the case of non-uniform motions. Nicholson then argued that Larmor's electromagnetic theory already showed that the principle of relativity did not extend to rotating systems.

2. Relativity at Cambridge from a logical point of view (1): Russell and Campbell

Britain's leading philosopher of logic and mathematics was Bertrand Russell, Fellow of Trinity College. Russell pursued more radically than Frege did in Germany, the project of logicism stated in the opening of *The Principles of Mathematics* (Russell, 1903: v): 'the proof that all pure mathematics deals exclusively with concepts definable in terms of a very small number of logical concepts, and that all its propositions are deducible from a very small number of fundamental logical principles.' In the same Preface he acknowledged that it was a number of conceptual problems about dynamics that had led him to questions in geometry and arithmetic. But the assumption underlying the connection was that dynamics and geometry were first rational parts of pure mathematics, and ultimately of logic, and only subsequently they were empirically applicable (Russell, 1903: vii).

Norman Campbell (1880-1949) entered Trinity College, Cambridge, where he received his undergraduate education with a University BA in 1902, Honors in the Mathematical Tripos and a College Fellowship in 1904. He conducted his experimental researches at the Cavendish Laboratory during the period 1902-1910. During the following four years he was the Cavendish Research Fellow in physics in the department at the University of Leeds, until the war broke out and he joined the National Physical Laboratory.

In contrast with his experimental researches on radioactivity, in 1910 he published his first article on the theoretical status of the concepts of absolute and relative motion. Its title, 'The Principles of Dynamics,' and focus echoed those of Russell's *The Principles of Mathematics*. The book had been published a year before Campbell became a fellow of Trinity College, his own undergraduate College, where Russell was already a fellow. In fact, Campbell decided then to start writing on philosophical issues related to science, an effort that led to a book manuscript titled after Euclid's book on axiomatic geometry, *Physics. The Elements*, published immediately after the war (Campbell, 1919). Campbell began the article by referring precisely to Russell's book and the defense it contained of the concept of absolute motion (Campbell, 1910a: 169).

Next he distinguished between theoretical and empirical dynamics and their respective problems with a focus on the usefulness of mathematics and the fundamental propositions he also called 'postulates of dynamics.' (Campbell, 1910a: 172). The postulational language loosely borrowed from Russell would become explicit and consistent in his discussion of relativity after reading Lewis and Tolman on relativity theory.[4] The logical axiomatic tone is explicit not only in the postulational language. The problem of theoretical dynamics, which Campbell expressed in three questions, is this: 'In what manner are the fundamental propositions from which the conclusions of theoretical dynamics are deduced to be stated; what are the conceptions employed in those propositions, and what are the relations stated between them?' (Campbell, 1910a: 169). The empirical problem is derivative and consists, instead, in the application of those propositions to experiment and the measurement of the magnitudes associate with their concepts. Pointing to Russell again, Campbell kept up the logical tone of the discussion with a logical principle for the identification of concepts. Two concepts are identical only 'if their definitions can be shown to be logically equivalent' (Campbell, 1910a: 170).

As Campbell had done in the previous discussions, in 'The Common Sense of Relativity' he focused on physical theory, which he defined as 'a set of fundamental propositions from which experimental laws may be *logicall*y deduced.' (Campbell, 1911a: 502; my emphasis). Despite emphasizing the experimental content, his analysis relied on a Russellian approach, adopting conceptual and logical standpoints to track logical deductions and eliminate 'fallacious' and 'apparently paradoxical' conclusions, 'confusions of thought' and 'misapprehensions.' (Campbell, 1911a: 502).[5] The source of the confusions, according to Campbell in his strategy already deployed in the article of dynamics, laid in the meaning of the terms 'velocity' and 'reality', in other words, in the logical inequivalence between the relativistic concepts and the ones adopted by the classical, Newtonian physicist.

This logical perspective is symbolized terminologically through the association with Einstein's principle of relativity –the standard denomination– of the axiomatic terminology of 'theory' and 'postulates.' That, in his own words,

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[4] Lewis and Tolman (1909) and, on the postulational American development of relativity, Cat (2016).

[5] In the 1919 book he noted that the propositions derived from the principle of relativity had been initially considered 'self-inconsistent and contradictory' (Campbell, 1919: 4 n.1).

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the 'Principle is what is more often termed a "theory"' might have been further suggested by the title of Sommerfeld's recent two-part article, 'Zur Relativitätstheorie.' (Sommerfeld, 1910a; 1910b). Campbell cited the second part in relation to the claim that Newtonian mechanics, as a fundamental theory, is inconsistent with the fundamental Principle of Relativity and must be abandoned as a result (Sommerfeld, 1910b: 684-89). The now-familiar term was introduced only in the title, whereas in the text Sommerfeld used throughout *Relativtheorie*. The postulational characterization of Einstein's two basic principles was suggested by Lewis and Tolman's paper and Tolman sequel, in a way consistent with Russell's axiomatic vocabulary.

Like Lewis and Tolman, Campbell expressed a logical preoccupation, namely, with the logical relation between the two postulates and their experimental implications (Lewis and Tolman, 1909; especially Tolman, 1910). Like them, especially Tolman, Campbell claimed that Einstein seemed to imply that the second postulate can be derived from the first alone, but, he argued, 'the Second Postulate cannot really be deduced from the First.' (Campbell, 1911a: 507). Also like Lewis and Tolman, Campbell saw in the theory of relativity an alternative to the Newtonian theory of mechanics; they raise a logical conflict: 'If they are not logically equivalent they must be contradictory; in either case an "explanation" of one in terms of the other is impossible.' (Campbell, 1911a: 516).

For Campbell, what was at stake was the logical clarity and consistency of theoretical dynamics and its distinction from empirical dynamics. For Lewis it was the formulation of a system of non-Newtonian mechanics based on general but exact principles of invariance or conservation. In their path to relativity, Lewis, Tolman and Campbell paid attention to empirical research and results in the nature of physical theory and in personal methodological practice, but their analyses featured the empirical only from a logical point of view they found embodied, prior to Einstein's theory, in ideals and standards of physical theory –dynamics for Campbell and thermodynamics for Lewis and Tolman–, of mathematics –geometry– and foundations of mathematics –Russell's and Hilbert's programs of axiomatics.

3. Relativity at Cambridge from a logical point of view (2): Enter Robb

Alfred Arthur Robb (1873-1936) began his studies at the Royal Belfast Academy and Queen's College, Belfast, graduating in 1894. Evidence of his early interest in Euclidean geometry is a problem-solution set he contributed in 1891 to the mathematics section of the *Educational Times Report*.[6] The contribution consisted in a method for dividing a circle into seven equal parts (the inscription of a heptagon in a circle by means of so-called Peaucellier cells), mentioned the following in a paper on geometrical approximations presented A.J. Pressland to the Edinburgh Mathematical Society (Pressland, 1892: 27).[7]

After graduating from Queen's College he was admitted at Cambridge and St. John's College the same year, graduating with a BA in 1897 receiving honors in the Mathematical Tripos.[8] Joseph Larmor had been his most admired teacher and probably recommended Robb for the opportunity to spend two years at the Cavendish as an advanced student, receiving an MA in 1901. This period was the heyday of electron research at the laboratory under Thomson's direction, with significant attention focused on the new kinds of radiation.

According to the Thomson's history of the Cavendish, Robb was affiliated with the laboratory during the 1906-1907 year (Thomson, 1910: 332). But his two publications during the pre-relativistic period 1905-1911 precede his official affiliation (Robb, 1905a and b). They suggest an earlier return to Cambridge, in 1905, reconnecting with local tradition of electron and ether theories, in particular, J.J.Thomson's researches in ionic physics. As in his dissertation, his approach now was neither experimental nor inductive; he focused on the formal conditions of solubility of a second-order differential equation and their physical interpretation. The equation, for the distribution of electric intensities as a function of the current flow and the velocities of positive and negative ions, had been introduced in 1899 by Thomson in his treatment of the conduction of electricity through gases between parallel plates. It lacks a general analytic solution, but he solved it for the case of equal velocities of negative and positive ions. Robb proposed a transformation that renders the equation a characteristic for any gas independent of the current and for

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6 *Educational Times Report* vol. LV, 61, question 10865.

7 Robb contributed another problem in 1908, the differential equation $d^2y/dx^2=Ay^mx^n$, see *Educational Times Report* March 2, 1908, 145, problem 16255.

8 Part I in 1897 and Part II in 1898.

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unequal velocities of the ions he identified additional solubility conditions, which he interpreted as two values of the ionic pressures –or velocity ratios (Robb, 1905a and b).

A literary composition of the same year offered a portrait of Cavendish electron research in relation to ether theory and electromagnetic radiation, also closer to his own dissertation work. It consisted of the lyrics for a song, in the manner of Maxwell's similar compositions, to be sung at the annual dinner of the laboratory's research students on December 6, and published in *Nature* (Robb, 1906). The song's title was 'The Revolution of the Corpuscle.' It began as follows:

> Air: "The Interfering Parrott." (Geisha.)
>
> A corpuscle once did oscillate so quickly to and fro,
>
> He always raised disturbances wherever he did go.
>
> He struggled hard for freedom against a powerful
>
> foe –
>
> An atom –who would not let him go.
>
> The aether trembled at his agitations
>
> In a manner so familiar that I only need to say,
>
> In accordance with Clerk Maxwell's six equations
>
> It tickled people's optics far away. (Robb, 1906)

It was also a period of electro-optical research in ether physics, conducted especially by Rayleigh, devising new ways to determine experimentally the velocity of light relative to the ether in the wake of Michelson's negative results. Robb attended Rayleigh's discussion of the results and its significance in ether theory at the meeting of the BAAA in 1902 (Robb, 1921: i). In 1903 Rayleigh announced another negative result based on polarization effects, extending the scope of the classical principle of relativity.

Robb succeeded in getting admitted to Göttingen in the winter semester of 1901 to pursue a doctorate under Woldermar Voigt, eventually writing a dissertation on the Zeeman effect (Robb, 1904). The effect was the splitting observed by Pieter Zeeman in spectroscopic absorption lines of atomic radiation in the presence of an external magnetic field. Its research linked together fields as diverse as chemistry, astronomy, mechanics, electron theory and radiation research. Voigt had three students specializing in different ways in physical optics, Robb, the Scottish R.A. Houstoun and Walther Ritz. Optics of moving bodies was a central problem for the group, a problem introduced by Voigt's own research.

4. Electron theory and the mechanical cone

In the wake of Michelson and Morley's null-result, in 1887 Voigt studied the transformations that electromagnetic waves in the elastic ether should satisfy to preserve their equations for observers in inertial motion. He derived a set of transformations (anticipating properties of Lorentz's) that preserved the velocity of light and included time dilation in moving clocks. The final application of this result was the propagation of wave surfaces in the shape of a light cone (Voigt 1887: 48-50).

In 1845 Faraday had detected a rotational effect magnetic fields had on the plane of polarization of light as it propagates through a crystal, but when in 1862 he tried to a magnetic field had on spectral lines he failed to find any effect. Subsequently, in 1877, Kerr had detected an effect on light reflected by magnetized iron (Lorentz, 1916: 98; Kox, 1997). Across the Channel, in Leiden, Lorentz's colleague Pieter Zeeman replicated Faraday's experiment in 1896, to detect a broadening of emission lines in sodium light. The same weekend of Zeeman's announcement Lorentz extended his ideas about electron theory to Zeeman's effect explaining it as a splitting of lines due to the oscillatory motion of bound electrons (atomic "ions") in the external magnetic field.

According to Lorentz, the oscillating charges moved according the harmonic law for atomic elastic (harmonic) forces and to his force law for external magnetic fields from 1892 that supplemented Maxwell's theory of electromagnetism. Three separate components of the oscillations –the three degrees of freedom corresponding to modes along different directions– could be detected in a direction perpendicular to the orientation of the field (this is the direction of the force exerted by a magnetic field on a moving charge). As a result, where one line could be observed corresponding to one frequency, now a triplet appeared.[9]

[9] As Lorentz put it, 'only a spectral line which consists of three coinciding lines can be changed into a triplet, the magnetic field producing no new lines, but only altering the positions of already existing ones.' (Lorentz, 1916: 112). The coincidence condition is expressed by the equivalence of three degrees of freedom, with equal corresponding frequencies. The explanation can be extended to quartets, quintets and other line multiplets. The physical origin of the triplets is encapsulated by this mathematical constraint.

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Then, at the turn of the century, new observations were reported. Zeeman published observations of further splitting of spectral lines into additional line multiplets, contradicting Michelson's recent interferometer observations. Voigt quickly formulated his own electron theory of the Zeeman effect. According to Voigt, the sets of lines linked to oscillating radiation were due to the vibrating motion of electrons bound by harmonic forces (Voigt, 1899).[10]

Robb followed Lorentz and Voigt in applying classical dynamics to explain Zeeman's sets of spectral lines in terms of the vibrating motion electrons. To explain additional multiplets such as quintets, he postulated a structure of the radiating particle containing a coupled pair of electrons in oscillating in response to an elastic central force. The positions and motions of both electrons are coordinated, respectively, by coupling constraints of a quadratic form (Robb, 1904).[11]

5. The axiomatic turn: physics and geometry

Robb's dissertation already introduced postulational elements of the axiomatic approach that would characterize his geometrical formulation of relativistic motion. The role of the coupling conditions in Robb's theory of the Zeeman effect is an instance of the role in theory-construction of basic propositions rather than inductive experimental generalizations. Indeed, in his introductory methodological remarks he endorsed the role of what he called fundamental assumptions (*Grundvoraussetzungen*) or postulates (*Postulate*):

> Bei dem Versuche, eine Theorie von irgend einem physikalischen Phänomen aufzustellen, muß man an gewisse Grundvoraussetzungen anknüpfen.
>
> Zuweilen geschieht es be idem Fortschritt der Wissenschaft, daß ältere Postulate sich zurückführen lassen auf andere, die einfacher zu sein scheinen; aber in jedem Falle ist dieses nur bis zu einem endlichen Grade möglich, und es kann sein, daß die Postulate, die schließlich erreicht warden, physikalische Grundwahrheiten sind, über die wir nicht hinausgehen können und die wir nur zu konstatieren vermögen. (Robb, 1904: 11)

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10 Voigt applied classical dynamics to the degrees of freedom parallel to the external field but coupled to the degrees of freedom corresponding to the radiating oscillations perpendicular to it. In the case of weak fields, with the theory Voigt predicted asymmetries in separation and intensity of the lines around a central component, already suggested by Zeeman, and immediately reported, for the intensity differences, by Zeeman himself, H.M. Reese and others.

11 In the chapter on the Zeeman effect in Lorentz's book on electron theory and its application to phenomena of electromagnetic radiation, Lorentz criticized the artificiality of Robb's coordination conditions (Lorentz, 1909).

The dominant standard of geometric theory might have become further validated by the experience of Göttingen culture; at the same time, the broader debates about axiomatics and the simplifying and revisable status of fundamental axioms such as the postulate of parallels might have suggested a less consistent terminological commitment to talk of axioms and the sort of qualification expressed by (inconsistent) talk of postulates.

Göttingen was the academic home to scientists whose researches set the course of mathematics and physics on a new path: Hilbert, the foremost proponent of the new axiomatic foundations of geometry (after his predecessor Riemann) and physics (Corry, 2004), Minkowski, the mathematical physicist of recent arrival from Zurich (in 1902) who would become interested in Lorentz's electron theory and would develop the four-dimensional geometrical reformulation that would eliminate some of the physical perplexities,[12] and Abraham, the foremost proponent of the new electron theory and the electromagnetic worldview (also Wiechert, Kaufmann, Schwarzchild, Drude and Sommerfeld worked on electron theory or its testing).[13]

Hilbert was entertaining the application of the axiomatization of mature theories to classical physics. Robb, electron theorist and geometrician, would have noticed. Most faculty mentioned above were members of Klein's mathematical physics seminar devoted to interdisciplinarity and the unification of physics. In an autobiographical sketch appended to his dissertation, Robb listed the names of professors whose lectures he had attended: Abraham, Hilbert, Klein, Manchot, Minkowski, Nernst, Riecke, Voigt and Wallach (Robb, 1904: 'Lebenslauf').

Klein placed mathematical physics between pure mathematical research and technical industrial application, fostering both. Mathematics played however a central unifying role. Nernst headed a newly created institute for physical chemistry; Nernst shared with Ostwald, at Leipzig, a systematic approach to energy physics (thermodynamics) from first principles, especially as the foundation of chemistry (the new discipline of physical chemistry). Also mechanics and electron theory were perceived and pursued as projects within the umbrella program of unification (this is the focus of theoretical physics at the core of Wien's unified electromagnetic worldview, based on physical hypotheses rather than mathematical structures). In his lectures on mechanics, Minkowski joined Hilbert in defending the axiomatic structure of

[12] See Minkowski (1909/1911), Miller (1981), Walter (1999) and Darrigol (2000).

[13] See Pyenson (1976) and (1979) and Miller (1981).

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mathematical and mechanical theories, with an emphasis, like Riemann's and Helmholtz's, on the origin of the axioms in physical facts and experiences (Pyenson, 1979: 66).

One of the interdisciplinary seminars in mathematical physics focused on electron theory, through the works on electricity and optics by Hertz and especially Poincaré, on electrodynamics by Larmor, Hertz, Lorentz and Poincaré and works by local faculty research on electron theory and the optics of moving bodies. The leaders of the 1905 electron theory seminar, Hilbert, Minkowski, Wiechert and Helglotz stand out among their Göttingen colleagues as the sole ones who became interested in relativity theory.

Younger faculty such as Born and Sommerfeld and students such as Robb, Ritz, Laub, Schlick and von Laue would also address relativity theory. Ritz developed an emission theory of light particles as an alternative to Einstein's second principle and the velocity-dependence of mass, Laub would collaborate with Einstein on the concept of force with a critique of Minkowski's dynamical assumptions and on the relativistic explanation of H.A. Wilson's effect, the polarization of a dielectric rotating in a magnetic field. Laub subsequently publish his attempt at a relativistic dispersion theory and a widely-cited comprehensive discussion of experimental foundations of the theory and von Laue penned the first book-length treatment of the theory (Laub, 1910; von Laue, 1911).

As Robb subsequently noted, his interest in relativity was rooted in Cambridge researches by Larmor and Rayleigh on the problem of the detection of effects of uniform motion in the ether (Robb, 1921: Preface). He became reacquainted with the issues from the Cambridge perspective at from statements by Rayleigh and Larmor at the 1902 meting of the British Association for the Advancement of Science in his hometown of Belfast, while he was already at Göttingen engaged in physical optics and electron theory in the Zeeman effect (at the meeting Robb presented a paper on the theory of determinants). Researches at Cambridge and Göttingen are the roots of Robb's combined interests in relativity theory and the physical basis of the relevant geometric concepts such as equality of length, which he revisited after becoming acquainted with Einstein and Minkowski's geometrical and kinematic principles.

During the decade that followed Robb's Göttingen work, I will argue, his interests in geometry and the relation between mechanics and optics –in his dissertation, between the motion of electrons and their radiation– underwent a dual transformation that may be characterized as (1) a synthesis and, more specifically, (2) an axiomatic and logical turn. This turn was more explicit and systematic than I have identified, above, in Campbell's work. In particular, what became understood as his version of relativity theory was the result of (1) addressing a few problems similar to Einstein's and (2) the project of solving them by providing an axiomatic, geometric foundation of physics by means of an axiomatic, physical foundation of geometry.

To understand this systematic development, one should look to (1) the publication Einstein's special theory of relativity and Minkowski's formulation in terms of a four-dimensional geometry of space-time considered, with Robb, as contributions to physical geometry,[14] and, I suggest, also to (2) Robb's education in the new developments in geometry and his selective engagement and eclectic mix –a further synthesis– of different strands among the new trends: Helmholtz, Poincaré, Hilbert, the Italian school, Russell and the so-called American postulationists.

Robb's commitment to axiomatization was informed by an interest in logical properties –the logical structure of theory–, but not in logicism –the logical foundation of theory. Each was related to a different tradition of mathematical logic, one of mathematics in logic (ex., Boole, Schröder, Peano) and the other, more recent, of logic in mathematics (ex., Bolzano, Frege, Russell).[15]

In the axiomatic foundations of mathematics, two different epistemological perspectives corresponded with different locations for the epistemic grounds for those theories. From the perspective I call externalist, the grounds were external to the system, and the abstract character of the basic propositions and their concepts did not show any relation to those origins or motivations or even justifications; this approach was shared by Riemann, Pasch, Hilbert, the Italian School and Poincaré. From the other, internalist, perspective, new foundations were expressed by new axioms; this was shared by Helmholtz, Russell, Frege and Robb.

Rather than taking place in the context of the Cavendish research, the particular evolution of Robb's work might have been facilitated by the membership in the more mathematical orientation of the Cambridge Philosophical Society and the London Mathematical Society (to which he was elected in 1905 and 1910, respectively).

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[14] The more obvious physical precursor works in relativity and in geometry available to Robb, including Russell and Whitehead at Cambridge, have been briefly noted in Sánchez Ron (1987), Walter (1999) and Barrow-Green and Gray (2006).

[15] Mancosu notes the prevalence of this distinction at Göttingen in Mancosu (2003).

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Outside the Cavendish, Cambridge offered Robb access to Russell and his work on the logical foundations of mathematics, including geometry. In the wake of Einstein's publications, which like so many colleagues, Robb would have related to his work in electron theory, he also followed theoretical developments in Göttingen, especially by Minkowski. The principled formulation of Einstein's theory in 1905 and Minkowski's four-dimensional space-time geometric rendition of 1908 fitted with the logical and axiomatic elements in the foundations of geometry (and mathematics). Robb didn't separate neatly the two projects; one served the other.

Minkowski introduced his non-Euclidean geometric account of the kinematics of relativity theory within the tradition of evolving axiomatic formulations of geometry and mechanics. He worked in the footsteps of his friend Hilbert, who had laid out a project of axiomatization of physics in lectures during 1905 and at a seminar on electron theory jointly convened by the two of them with the participation of other faculty and a seminar on electrodynamics led by both (Corry, 1997). For Hilbert and Minkowski the application of the axiomatic method served the epistemic function of identifying and evaluating physical and mathematical assumptions, including definitions implied in empirical claims and mathematically proven results (Corry, 1997: 280). Considering, as most scientists did, that Einstein's paper of 1905 was a contribution to electrodynamics and electron theory, Minkowski asked Einstein for a copy for discussion at the 1905 seminar. He considered it from the point of view of Hilbert's program and his own mathematical interests, in this case geometry. The result helped articulate the radical implications of Einstein's proposal and to arouse wider interest among theoretical physicists and mathematicians.

In his now classic talk from 1908 'Raum und Zeit' Minkowski formulated its central idea in the form of a fundamental postulate, namely, of the positive value of the space-time interval, which he interpreted kinematically as the limiting value of the speed of light. Geometrically, it introduced an absolute perspective in physical geometry that recognized independent reality only in four-dimensional space-time (Minkowski, 1909/1911). Physical geometry is a temporal geometry. Minkowski went beyond the mere juxtaposition of separate axioms of geometry and mechanics that Hilbert had endorsed in 1905 in a lecture on the axiomatization of laws of motion (Corry, 1997: 293). For Hilbert the analysis of motion required only adding axioms about two basic properties of time, its uniform passage and unidimensionality. In his new, integrated framework, Minkowski represented the two-dimensional invariant relation between x and t, that is, $ct^2-x^2=1$, with a hyperbolic curve. The curved also expressed the transformations of subluminal uniform velocities bounded by the absolute value of c –the Lorentz group of transformations Minkowski now interpreted as a group of rotations in four-dimensional space-time. The curve is contained within the geometric form of a cone of light signals with an absolute velocity. The null interval provides the equation for the geometric figure of the cone. The negative values of the time variable defined the fore-cone (*Vorkegel*) –that is, the past cone– and the positive value the aft-cone (*Nachkegel*) –that is, the future cone (Minkowski 1909/1911: vol. 2, 438).

He presented his condition on the space-time interval as the space-time version of Einstein relativistic group-invariance principle and called it first an axiom, next he suggested 'Relativity Postulate' (*Relativitätspostulat*) but rejected it as 'weak' since it appeared limited to the manifestation of four-dimensional space-time by phenomena; finally, in his own terms, he called it the Postulate of the Absolute World, or World-Postulate, for short (*Weltpostulat*) (Minkowski 1909/1911: vol. 2, 437). In an earlier talk aiming to include dynamics, electromagnetism and gravitation, Minkowski detailed how the equations of electrodynamics of moving bodies would follow from the postulate and two other axioms about transformations of quantities and equations relating them (Minkowski, 1915).[16] In line with Hilbert's program, Minkowski called them axioms. The so-called postulate didn't just adopt, like Poincaré had done, the general invariance of mechanics and electrodynamics as an inductive generalization about the absence of absolute rest among the properties of observable phenomena. For Minkowski relativity was a normative principle, a constraint on the construction or discovery of new physical theories of observable phenomena. Needless to say, postulate-talk is hardly Hilbert's and resonates with the vocabulary of classic Euclidean geometry and Italians' new approach to axiomatics (see below). It was just not Einstein's principle of relativity, whose designation Minkowski used for the principle of covariance of physical laws under Lorentz transformations.

Göttingen was not the source exclusively of Hilbert-style, formal axiomatics. Robb's education in electron theory alone, if not also in geometry, would have led him to Helmholtz and Poincaré, who had written both on electromagnetism and geometry. Russell's would work too (including his exchanges with Poincaré). The diversity of doctrines, then, would have led him to adopt, broadly speaking, a foundational, axiomatic perspective with an emphasis on logical considerations of demonstration and of consistency and independence

[16] The talk, from 1907, was published posthumously.

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of axioms. Then he resolved the issue of which specific doctrine to adopt on terms set by his own particular dynamical model of radiating electrons and the more fundamental kinematic perspective of relativity theory.[17]

In the context of the Göttingen interdisciplinary seminar, Hilbert projected extending the mathematical standard of axiomatics to mathematical physics, by appealing of first physical principles and concepts. But his axiomatization of geometry followed earlier German formulations by Pasch and formulations by Peano and his followers in the Italian school such as Padoa, Pieri and Veronese in leaving his symbolic axioms abstract. In this way they remained independent of any interpretation, except internal, formal implicit definitions of the concepts, in Veronese's analogy, like roots of a system of logical equations. Consistency itself was a formal property that determined the existence of mathematical entities and the truth of propositions about them. From an externalist point of view, the axioms provided, as axioms of geometry, a logical analysis of human spatial intuitions, stating, as he put it, fundamental facts about them (Hilbert, 1899). In general, however, Hilbert's formalistic approach to axiomatics constituted and was most influential as what has been called an image –a standard or ideal– rather than a body of mathematical knowledge.[18]

6. Axiomatics and physical geometry

While Russell and Whitehead were publishing at Cambridge axiomatic systems of geometry and defending their famous reductive project of logicism, Robb engaged, instead, the empiricist physical approach he encountered in Helmholtz. This earlier German doctrine on the foundations of geometry was more in fitting with Robb's commitment to dynamical description as well as the quasi-axiomatic and geometric formulations of relativity theory.[19] It was Riemann at Göttingen who had first pitted non-Euclidean geometry against the Kantian idealist doctrine on geometry and famously declared that geometry's foundations were constituted by hypotheses.

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[17] Walter has offered a brief examination of the latter as an instance of non-Euclidean trend in relativity theory that included Minkowski, Sommerfeld and Robb; see Walter (1999). Here I emphasize, instead, the evolving synthesis of logical and physical aspects of Robb's interest in axiomatics in physical geometry.

[18] On the distinction see Corry (2004).

[19] On the interpretation of Helmholtz's philosophy of geometry I follow Torretti (1978) and DiSalle (1993).

Not unlike Helmholtz, and then partly in Robb, Riemann's epistemological approach to geometry rested on his background in experimentation and the application of analysis in physics, especially in ether physics –in relation to electric and magnetic forces– end energy physics (Ferreirós, 2006; Torretti 1978). At Göttingen he had developed interests in electricity and geometry alongside his teachers Gauss and Wilhelm Weber, who had collaborated on empirical determination of units of measurement in geometry and electromagnetism. Riemann's so-called problem of space, then, consisted in identifying the empirical hypotheses that singled out Euclidean geometry. From what I have called an externalist perspective, the hypotheses were traceable to the physical experience of space, the possibility of empirical measurement in it and the related motion of objects.[20]

Helmholtz sought to ground Riemann's geometrical hypotheses, internally, on axioms describing more fundamental empirical facts. The empirical character of the fundamental facts was linked to Helmholtz's ideas about the origins of visual space in anatomy, physiology and learning habits. Learning from the experience of physical objects and developing 'intuitions of fixed typical relations' was the general context for more specific scientific facts, the mechanical conditions of geometrical measurements. The fundamental grounds for geometry concerned the free mobility of rigid objects required for the comparison of spatial magnitudes. While the factual grounds challenged the Kantian idealist doctrine of the a priori status of Euclidean geometry, in this context and in the foundation of physics on energy conservation Helmholtz retained Kant's causal standard of intelligibility of Nature.[21]

In this tradition, Minkowski's non-Euclidean geometry of space-time was the latest significant contribution.

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[20] Riemann's five hypotheses, simplified, are the following: (1) space –that is, the concept of geometrical space– is a continuous differentiable manifold of n-dimensions (a set of points whose n-coordinates were the result of measurements); (2) space has 3 dimensions; (3) space allows for the definition of an element of length in the form of the square root of a quadratic differential expression of the coordinate functions (metric); (4) the curvature of space is constant; (5) space has constant zero curvature (flatness).

[21] The axioms at the basis of the system of Euclidean geometry are the following: (1) space is a n-dim manifold whose points are characterized by the n coordinates with values resulting from the measurements of n continuous differentiable variables (non-factual definition that enables the application of analysis in geometry and physics); (2) rigid bodies exist in space characterized by movable point pairs whose relations are unaffected by the body's motion (rigidity is an empirical idealization); (3) rigid bodies can move freely to any points in space; (4) the rotation of any rigid bodies can take it, without reversal of motion, back to its initial position; and (5) space has 3 dimensions.

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7. Russell's logical point of view: philosophy of mathematics between philosophy of logic and philosophy of language

In relation to Riemann's axioms, or hypotheses, Helmholtz intended his third axiom to prove Riemann's fourth, on the constant curvature of space. Subsequent authors such as Peano, Padoa and Russell introduced more abstract, purely kinematic versions, namely, de-materizalized, acausal conditions of free mobility without distortion of spatial magnitudes in terms of mathematical transformations. From their point of view, especially Russell's, geometry preceded dynamics.

Russell's philosophy of mathematics, logic and knowledge more generally developed in reaction to fundamental assumptions and claims of idealist and rationalist doctrines encountered at Cambridge and Oxford (his references and target included both Kant and Leibniz). The character of the engagement changed from being critical but sympathetic to being by the end of the century outright dismissive.

The gradual rejection of idealism required a conceptually coherent theory of geometry. The coherence of geometry, and mathematics more generally, relied on its reduction to concepts of numbers and next to concepts of logic. Although for Russell, axiomatics reflected the conception of geometry as a branch of logic, he also defended, like Frege, an empirical interpretation of the origins of geometrical concepts and principles, especially Euclidean (unlike in Hilbert's presentation); the doctrine provided the model of his epistemology of atomic sensations as the basis for the logical construction of theoretical entities (his solution to the problem of the knowledge of the external world that inspired Carnap). Similar views had been shared by Mach and Poincaré's, who also defended a conventional approach to the axiom basis admitting of empirical and mechanical explanations (defended also by Riemann and Helmholtz).

The new rigorous mathematics and mathematical logic showed the way. This Russell learned from German mathematicians such as Bolzano, Dedekind, Weierstrass, Cantor and Pasch and, especially, from Peano. Russell heard Peano first in 1897 at the Congress of Mathematicians held in Zurich, where Peano presented his symbolic notation for a mathematical logic, and then in Paris in 1900 at the Congress of Philosophy, where Peano presented a logic of relations and undefinable terms. This doctrine shared with Leibniz's project of characteristic universalis the ideal of a universal symbolic language on which philosophical method could rely to solve philosophical problems. It also contrasted with Hilbert's formulation of axiomatics; and yet both yielded a modern new image of mathematics, a standard established at the new foundational space between mathematics and philosophy.

From a symbolic standpoint, the generality of logic and mathematics was based on unrestricted character of variables (concepts). But then every proposition would, on his realist metaphysics, contain every entity; this cannot be a condition of actual knowledge and its boundaries, since it challenges the boundary of the mind's grasp, which excludes the infinitely large or complex, and also the boundary between logic and non-logic as matter of generality of scope.

Here problems of philosophy of language and mathematics dovetailed. Terms in propositions cannot denote without restrictions.[22] Similarly, numbers cannot be just classes of similar classes of objects. In both cases Russell encountered famous paradoxes and inconsistencies. His solution involved distinctions between logical levels. In the mathematical case of numbers, the distinction between functions and their arguments introduce levels of types. In the linguistic case, the denoting concept or term cannot be part of the content of the proposition. On a related basis Russell developed a general theory of descriptions and of knowledge, in which he famously distinguished knowledge by acquaintance of elementary constituents –whether logical or perceptual– and knowledge by description of higher-level constructs.

In mathematics, his rejection of idealism rested on the defense of the reductionist program of logicism, known then as logistics. The first formulation of the program along with discussion of the issues in metaphysics and language appeared in *Principles of Geometry* (Russell, 1903). There he followed on the steps of Peano and others in the so-called Italian school. They shared the new symbolic, logical preoccupation with axiomatizing geometry and arithmetic; but they differed in the epistemic assumptions about the status, interpretation and sources of the axioms.

In Russell's case, the particular ambivalent stance towards the basic propositions of geometry received linguistic expression in the interchangeable use of the terms 'axioms' and 'postulates' in relation to the revisable axioms of parallels in Euclidean geometry (for different reasons, the same stance and expressions are apparent in Poincaré's views). This epistemic lack of consensus was pushed to the limit by challenges raised by Poincaré's convention-

22 On denoting see Russell (1903) and (1905b).

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alism and American mathematicians' postulational philosophy. It challenged the notion that any view was necessary and opened a domain of freedom of exploration, like non-Euclidean geometries had done in their specific domain.

In order to pursue the reduction of geometry to numbers and then to logic, Russell offered a comprehensive study of different geometries, descriptive, projective and metric. He distinguished between pure and applied geometry; the former studied hypothetical properties of abstract structures, which in the latter are studied as representations of the properties of space in the actual world. In the latter he identified three axioms reminiscent of Riemann's and Helmholtz's empirical solutions of the so-called problem of space: the axiom of dimension, the axiom of distance and the axiom of free mobility (Russell, 1899; Torretti, 1978). His ambition however aimed at a more general understanding of geometry, in relation to the standards set by earlier projects of a rigorous arithmetization of analysis and the formal organization and classification of different branches of mathematics, whether in terms of the algebra of group theory or of logic of axiomatization. While Poincaré was declaring that geometry is the study of certain groups, Russell declared that 'geometry is the study of series of one or more dimensions.' (Russell, 1903: 372).

8. Robb's logical turn and the application of new axiomatics

Unlike Russell, Robb was a physicist. He was interested in principled approaches in physics and the geometrical formulations of relativity theory raised the more specific issue of the application of geometry in physics. Already familiar with Hilbert's program of axiomatics in mathematics and physics, he read into Russell's new ideas the importance of the preoccupation with logic, the formal standard set by the Italians, the shifting status of interpretation and motivation of axioms and the post-Riemannian, abstract kinematic axioms of geometry. These assumptions were compatible with his project of physical geometry and were independent of Russell's particular project of logicism, for which Robb had no use. Still, Russell's terminology of logic, axioms/postulates, terms, order, series and classes found physical application in Robb's axiomatic project of 1911 and 1914.

Pursuing his interests in physical geometry and the logical aspects of axiomatics, Robb remained engaged in ongoing foundational debates. In the process, he encountered and rejected two opposite doctrines: Poincaré's conventionalism and Russell's logicism. Between logicism, idealism and Helmholtz's physical empiricism, Poincaré had defended conventionalism. Robb would have been familiar with Poincaré's work in relation to two active subjects of research at Göttingen, geometry and electron theory. According to Poincaré, the lesson from non-Euclidean systems of geometry was that geometric axioms were convenient conventions, linked to ideas from human experience but not necessitated by it (Poincaré, 1902). The preference for the Euclidean system of geometry combined elements of ancestral adaptation of the human species and elements of choice based on the exercise of the will in scientific contexts involving practices such as measurement and theorizing. Unlike for the case of arithmetic, in geometry Poincaré sought to avoid what he took to be pitfalls of overly limiting Kantian idealism and Mill's naïve empiricism. Poincaré's was an account of the foundations of geometry but, like Riemann and Helmholtz's, it was an anti-foundationalist one, rejecting the old notion of axioms providing firm foundations, as either self-evident or universal and necessary propositions. The necessity of disguised definitions is as illusory as the stability of their geometrical facts and hypotheses.

Robb's more general interest in logical and mathematical matters relating to Russell's ideas he pursued briefly and privately, at least in correspondence with Russell himself. Upon his return to Cambridge, Robb was elected to the Cambridge Philosophical Society (November 1905) and subsequently to the London Mathematical Society, where he presented, inter alia, some of the ideas that developed into his *Optical Geometry of Motion*. It must have been during this period that he became further acquainted with work in foundations of mathematics such as Russell's in, for instance, *The Principles of Mathematics* (1903).

In 1908 and 1914 Robb engaged in a brief correspondence with Russell.[23] It began with a letter to Russell of 29 August, 1906, which Russell labeled 'from A.A. Robb, a distinguished geometer' and annotated indicating that Robb had applied himself to the contradiction, or Russell's paradox, in the theory of classes (Russell, 1903)–in fact, also contradictions discussed in the treatment of definite descriptions (Russell, 1905b). In a letter of 5 February 1908, he complained to Russell that the paper had been rejected by *Mind*, on a recommendation from a referee designated as 'R.' Robb would write a paper on the subject of the contradiction in terms of differences of definitions of terms involved (sameness of extensions) and preventing self-referential or reflexive increases of extensions applicable, per Russell's request, also to the

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[23] Bertrand Russell Archives, McMaster University, documents: 2637/054860/5.39/RA1 710, 79662/054861/5.39/RA1 710, 79663/054862/5.39/RA1 710, 79664/054863/5.39/RA1 710, 79665/054864/5.39/RA1 710, 79666/054865/5.39/RA1 710, 75368/250221/7.28/RA3 1027 and 79668/0548666/5.39/RA1 710.

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liar's paradox (letter of 11 February). He also used the example of the law of non-contradiction to illustrate the idea that propositions do not refer to themselves.

In the Cambridge physical tradition (traceable to Maxwell's molecular model of the mechanical ether), Robb modeled an argument or train of thought in terms of closed connected systems of cogwheels that might prevent connected motion unless in even number, illustrating the different ways to identify and resolve a paradox or contradiction (the different possible unwarranted assumptions), the logical counterpart of the mechanical problem (letter of 15 February). Similarly, in *Optical Geometry of Motion*, the elimination of apparent contradiction resulting from non-uniqueness of relative time in the theory of relativity is removed by the right representation of the causal connection between particles.

Robb pursued a defense of his paper on contradiction in three other letters of the same month and an acrimonious letter of March 1st objects to Russell's unwillingness to correspond on the matter. Nevertheless, Robb would send Russell a copy of his manuscript of *A Theory of Time and Space* and in a letter of 18 January 1914 acknowledged Russell's favorable report on the manuscript for publication by Cambridge University Press.

Robb introduced his treatment of relativity from a logical point of view: 'This remarkable suggestion [relativity of simultaneity] was at once seized upon with it apparently not being noticed that it struck at the very foundation of logic. That a thing cannot 'be and not be at the same time' has long been accepted as one of the first principles of reasoning, but there it appeared for the first time in science to be definitely laid aside.' (Robb 1914b, 2).[24]

The correspondence evinces Robb's interest and reputation in the foundations of geometry and Russell's work more broadly, which was based on axiomatic and hierarchical structures and the philosophical use of formal logical language and the constructive techniques of formal logic. The correspondence also reflects Robb's attention to the journal *Mind*, where Russell had published significant essays, including 'On Denoting', with a discussion of a paradox in the philosophy of language and the application of logic to prevent it (Russell, 1905b). In 1905 Russell had also published an unfavorable review of the new English edition of Poincaré's *Science and Hypothesis* (with

[24] Goldberg has noted Robb's reaction to relativity theory as resting on a logical fallacy, although only as an example of British physicists' failure to understand relativity; the new wine, to follow his analogy about the reception of revolutions, was kept in old bottles that left it in the dark; see Goldberg (1984: 234).

a preface by Robb's mentor Larmor) criticizing the conventional, rather than the experimental, view of principles of geometry and mechanics (Russell, 1905a).

While Robb rejected Poincaré's conventionalism explicitly in 1911 in *Optical Geometry of Motion* (Robb, 1911), Russell's philosophy he ignored. In Russell he nevertheless might have learned about related works from the Italian school that had also informed Hilbert's project of axiomatics. By 1914 he had subsequently incorporated related insights from Russell, the Italians and the American school, integrating them into the development of the 1911 work, *A Theory of Time and Space* (Robb, 1914b). This time Russell not only received the work favorably; he also adopted its key earlier ideas in his new discussion of empirical theory of knowledge of the same year (Russell, 1914).

Robb's engagement of logic, axiomatics and related issues in the foundations of geometry shifted noticeably between 1904 and 1911 and after. By 1911 Robb's new work on physical geometry had shown a new though relatively limited engagement (and acquaintance) with the debates in geometry and new positions in foundations of geometry and axiomatics, not even Russell's. Readings of the relevant works, especially by members of the Italian School, may have been prompted much earlier by his own interest in the field, Russell's own work, and even the more traditional specialized papers at the Mathematical Society. They may have been prompted also by reading Veblen sometime after 1911, which includes references to Italians also in Russell's *Principles*, and to fellow Americans Young and Huntington. 1910-1913 is also the period of publication of the three volumes of Russell and Whitehead's *Principia Mathematica*. In the Preface to the first volume, Russell and Whitehead declare that in geometry 'we have had continually before us the writings of v. Staudt, Pasch, Peano, Pieri, and Veblen.' (Whitehead & Russell, 1910: vol. 1, ix).

9. Robb's Italian and American sources

Besides German and British sources, we should mention the Italian and American. They combined distinctive approaches to the foundations of mathematics from an axiomatic point of view a marked . The so-called Italian school, around Veronese, Peano, Padoa and Pieri, broke ground with an uncompromising and systematic effort to formulate logical systems of arithmetic and geometry. Veronese's *Fondamenti di Geometria* (1891) referred to Euclid's postulates, otherwise for the general new foundations he mentioned axioms (*assiomi*). Peano, concerned with universal artificial languages for

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precise mathematical concepts and proofs, looked to Grassmann and Pasch, and wrote in terms of fundamental propositions (*proposizioni fundamentali*), which he divided into definitions (*definizioni*) and primitive propositions or axioms (*proposizioni primitivi* o *assiomi*). He also referred to the fundamental or primitive propositions as, equivalently, 'axioms or postulates' ('*assiomi o postulati*'), but the main operative terms were 'definition', 'axiom' and 'theorem, especially in *I Principii di Geometria Logicamente Esposti* (1889).

Padoa similarly declared the purpose of formulating 'a system of unproven propositions (postulates) from which (from definitions and axioms) one could prove all other propositions.' (Padoa, 1904)[25]. Members of the Italian school such as Peano believed, despite the symbolic nature of the languages employed, in the perceptual or empirical origin of the propositions represented by the axioms: 'if one wants to give this work the name of geometry it is necessary that such hypotheses or postulates express the result of simple and elementary observations of physical figures.'[26]

The main exception to the widespread consideration of axioms in the first decade of the 20th century would be Poincaré and the so-called American postulate theorists. Poincaré referred to the parallels postulate as Euclid's postulate and more generally to the postulates and axioms of geometry in *Science and Hypothesis* (1902) and his review of Hilbert (1902), translated by Huntington (1903). Huntington preserved Poincaré's separate use, adopting a pre-axiomatics standard use, maybe apparently indicating the historical change in the status of Euclidean geometry and therefore the revisability status of its postulates; and, more generally, the conventional nature of all geometrical axioms. Poincaré formulated his theory of relativity precisely with an emphasis on the formal general elements such as the principle of relativity and the groups of transformations. Unlike Einstein, however, he believed in the empirical, inductive origin of the first principles.[27]

The so-called American postulate theorists such as Huntington, Veblen and Young, who often recover not just the disjunction 'axiom, or postulate', but make primary use of the term 'postulate' (most notably by Huntington) or else introduce alternatives such as 'assumption' (by Veblen and Young).[28]

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[25] Presented at the Paris Second International Congress of Mathematics 1900.

[26] Quoted in Corry (2004: 44); see also Torretti (1978), comp. Schlimm (2003).

[27] By contrast, his a priori commitments were the concepts of electron and ether; see Goldberg (1984).

[28] The rubric of American postulationists was introduced in Corcoran (1980) and examined further in Scanlan (1991). My account of American postulationism goes beyond the characterization in Scanlan (1991).

In a presentation to the American Mathematical Society of results from his doctoral dissertation, Huntington offered what he called 'a complete logical basis for a deductive mathematical theory' of absolute continuous magnitudes (Huntington, 1902).[29] The logical basis of the theory from which the relevant theorems are deduced is formed by six propositions he called postulates, with the following clarification:

> Following the usual distinction, we use "postulate" to mean a proposition the acceptance of which is demanded or agreed upon as a basis for future reasoning, reserving "axiom" to mean "a self-evident proposition, requiring no formal demonstration to prove its truth, but received and accepted as soon as mentioned. (Huntington, 1902: 264n)

The use of the term 'axiom' indicates, by contrast, the view that basic propositions express 'the essential characteristics' of the magnitude in question.[30]

We find further clarification of this attitude towards basic propositions in a logical treatment of mathematical theory in Veblen and Young's subsequent formulation of an axiomatic system of projective geometry. Echoing Hilbert, they wrote: 'The starting point of any strictly logical treatment of geometry (and indeed of any branch of mathematics) must be a set of undefined elements and relations, and a set of unproven propositions involving them.' (Veblen & Young 1910: vol. 1, 1; Veblen, 1903). Now, they explained that given the purely formal or abstract nature of the symbols, devoid of concrete content ('application or representation'), 'it is manifestly absurd to speak of a proposition involving these symbols as self-evident', thus 'the unproved propositions referred to above must be regarded as mere assumptions.' (Veblen & Young 1910: vol. 1, 2). Yet, their motivation differed from Huntington's and the customary use in ignoring the axiom/postulate distinction and attributing the absence of self-evidence less to the absence of essential information than to the formal status: 'It is customary to refer to these fundamental propositions as axioms or postulates, but we prefer to retain the term assumption as more expressive of their real logical character.' (Veblen & Young 1910: vol. 1, 2). Axioms or assumptions, in this logical sense, are characterized by their

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[29] His dissertation at Strassbourg in 1901 included a discussion of complex quantities.

[30] Ibid., 264; in his dissertation, Huntington refers to the Archimedes postulate and the postulate of continuity, but also refers to them as axioms and uses, more generally, the expression 'das Axiom oder Postulat'; Huntington (1901: xvi).

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status as unproven, in particular not being deduced from other such axioms, and fundamental in their joint capacity of the set of axioms to derive every proposition associated with, for instance, Euclidean geometry.

In 1911, Veblen again re-stated his original position:

> the first propositions of all cannot be deduced, because there are no previous propositions to deduce them from. There must therefore be *assumptions*. These may be stated so plausibly that no one doubts their truth, but whether they are true or not cannot affect the correctness of the reasoning based upon them, nor the fact that they are assumptions. We shall not enter into the metaphysical question as to whether these assumptions are self-evident truths, axioms, common notions, experimental data or what not, but shall try to keep within the realm of mathematics by using the non-committal word assumptions. (Veblen, 1911: 4-5, original italics)

Huntington distinguished axioms from postulates or assumptions as statements of fact, whereas the word 'postulate' recovers the classic meaning of unsettled demand that might or not be satisfied (Huntington, 1911: 171). Veblen added in a footnote that truth becomes a hypothetical matter of testable validity: 'the truth of a statement can be determined only by testing all its consequences, so that the real test of the validity of the hypotheses of geometry is the validity of the theorems.' (Veblen, 1911: 4n).

Veblen cast in the new terms or at least the new conception of axioms as basic statements the old Euclidean project of geometry:

> the problem of the foundations of geometry is to find a system of axioms that is necessary and sufficient for geometry, –necessary, in that no axiom can be dispensed with, and sufficient for the deduction of the whole system of geometrical knowledge. (Veblen, 1903: 309)

For Veblen and Huntington this is the standard that grants the status of science (Huntington, 1911: 158). The new standard included also the rejection of Euclidean geometry as the unique solution and Hilbert's new conception of mathematics as 'a body of propositions stated about certain elements in terms of certain relations.' (Veblen, 1903: 304). The new formalistic perspective championed by Hilbert was based on relations between uninterpreted symbols, 'devoid of content', and the derivation of theorems by the methods of formal logic. According to Veblen the new perspective implied the new postulational attitude:

> Since it is manifestly absurd to speak of a proposition involving these symbols as self-evident, the unproved propositions referred to above must be regarded as mere *assumptions*. It is customary to refer to these fundamental propositions as axioms or postulates, but we prefer to retain the term assumption as more expressive of their real logical character. (Veblen & Young, 1910: vol. 1, 1-2).

Huntington emphasized two connected aspects of postulates that suggest his distinctive commitment to postulate-based axiomatics was motivated also by his focus on algebra rather than geometry. Conditions expressed by postulates are lack propositional content, therefore they can't be self-evidently true because they lack truth-value altogether. To categorize them, Huntington borrowed Russell's notion of propositional function (Huntington, 1911: 172). Moreover, he appealed to the original meaning of the term 'postulate' as a demand, rather than a description, that systems might fail to meet. He gave the example of army admission conditions in the form of requirements or instructions that a class of men might not satisfy (Huntington, 1911: 172). This constructionist approach fits the case of algebraic systems. The relevant systems to each the postulates apply, that is, that are eligible for satisfying them, consist of a class of elements such as geometrical points or numbers, and relations they obey such as order or betweenness. One may think, instead, in a formal mode, of regulating the meaning of terms, 'points' and 'betweenness.' In the case of algebraic systems, the specific types of relations between elements in a system are rules or operations, which fit the character of demand or instruction that distinguished postulates.

In a review of subsequent work by Huntington, N.J. Lennes drew attention to this precise significance in the by now anomalous terminological choices; his diagnosis combined Huntington's terminology with Veblen's interpretation:

> The original name for the unproved propositions of a mathematical science was "axiom," –a truth so simple that everyone must assent to it whenever the statement is fully comprehended. In this respect, the point of view has changed completely. If a, b, (+), (.) are purely abstract symbols, then no propositions whatever is evident about them. Hence the word "axiom" with its old connotation is being discarded. The paper under review uses "postulate." Other writers, as Veblen and Young, are using "assumption". (Lennes, 1911: 364-5)

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Their terminological particularity adopted along with the focus on either postulates or axioms understood in new epistemological ways has been associated with a broader uniformity of interest and approach to foundational mathematical research.[31] The explicit rejection of the traditional foundationalist connotation of the term 'axiom', when not the introduction of the postulational rubric, marks changing commitments to axiomatics and to the logical and epistemological status of axioms. It is a commitment to foundations without foundationalism.

Their work emphasizes the distinction between the syntactic undefined expressions in the systems, or postulate sets, and their semantic interpretations. On the basis of the distinction Veblen and Huntington followed Hilbert in developing formal metatheoretical conditions of adequacy of the sets such as consistency, sufficiency and independence (or irreducibility), which together characterized the completeness of a set of postulates and granted them the status of logical basis for a deductive mathematical theory (Huntington, 1902: 264; Huntington 1911). With an explicit and systematic semantic approach missing in Hilbert's formulation of axiomatics, Veblen and Huntington articulated a model-theoretic conception of the metalogical categories in terms of satisfaction and truth, including the notion of equivalence of classes of objects satisfying the same sets of postulates, that is, 'that there is *essentially only one* class of which the twelve axioms are valid.' (Veblen, 1904: 346, original italics). Veblen introduced for this property the term 'categorical.'[32] Neither author used the word 'model.' Huntington described the equivalence between 'assemblages' in terms of isomorphism, or one-to-one correspondence; then he proves a theorem:

> any two assemblages M and M' which satisfy the postulates 1-6 are equivalent; that is, they can be brought into one-to-one correspondence in such a way that a o b will correspond with a' o b' whenever a and b in M correspond with a' and b' in M' respectively. (Huntington, 1902: 277)

[31] Corcoran (1980) and especially Scanlan (1991), although he glosses over the above distinctions. According to these proposals, members of the school of American postulationists include L.E. Dickson, E.H. Moore, E.V. Huntington, O. Veblen, J.W. Young, R.L. Moore, B.A. Bernstein, R.E. Hendrick, J. R. Klein, H.M. Sheffer, C.H. Langford and C.J. Keyser. Their locations are predominantly Chicago (E.H. Moore and his student Dickson), Princeton (Veblen) and Harvard (Huntington).

[32] According to Veblen, the term was suggested by John Dewey; ibid. Huntington introduced it in 1911 as the term of a sufficient set of postulates (Huntington, 1911: 171n).

Veblen subsequently called the objects satisfying a consistent set of assumptions their *interpretation* and, equivalently, their *concrete representation* or *application of an abstract science* (Veblen & Young, 1910: vol. 1, 3 and 336).

After the First World War postulationism was identified and defended as a doctrine in the philosophy of mathematics by Cassius Jackson Keyser at Columbia and, more generally, by D.R. Carmichael then at Illinois, Urbana-Champaign (Cat, 2016). Carmichael's commitment dates back to his prewar work on relativity.

10. Relativistic postulationism

Veblen's attitude towards assumptions is the view at work in Einstein's original choice of the term 'presuppositions' (*Vorausetzungen*) and the view adopted also by Carmichael in his postulate formulation of relativity theory (Carmichael, 1912; 1913). While familiar with the wok of Huntington and others, in relation to relativity, Carmichael borrowed the emphasis on postulates from Lewis and Tolman (1909; and Tolman 1910). So did, I believe, Einstein himself, von Laue and then others adopted too, at least in relation to the name and status of the basic principles becoming the two postulates of relativity.[33] By 1911 Einstein, had read G.N. Lewis and Richard Tolman's paper on the new non-Newtonian system of mechanics after receiving copies probably from Lewis himself, whom Einstein had met in Zürich in 1901. Also von Laue had read it as well as Tolman's follow-up paper and cited it in his textbook treatment of the theory, which Einstein read as well (von Laue, 1911). Einstein praised Lewis and Tolman's paper to Vladimir Varičak as a beautiful study.[34] Further proof of his re-acquaintance with Lewis' work was his recommendation to his collaborator Jakob Laub to contact Lewis for employment at the MIT laboratory of physical chemistry.[35]

In light of Einstein's physico-chemical researches it is not surprising that in 1901 in Zürich he received the visit of Ostwald's former assistant and colleague George Bredig and his American visitor G.N. Lewis.[36] Bredig was ap-

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[33] I offer a study of this development as part of a multidisciplinary history of relativity in America in Cat (2016).

[34] Einstein's letter of 3 March, 1911, CPAE vol. 5, doc. 257a, 7 (*The Collected Papers of Albert Einstein*, Princeton, NJ: Princeton university Press).

[35] Einstein's letter of 9 November, 1910, CPAE vol. 5, doc. 231, 166.

[36] Lewis' letters to Einstein, 23 February 1921 and 27 July 1926, CPAE vol. 12, doc 62, 100, and vol. 5,

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pointed to the ETH in 1910 and was Einstein's colleague there between 1912 and 1914. Like T.W. Richards, his Harvard advisor before him, Lewis had taken the opportunity to spend a year in Germany with Ostwald in Leipzig and Nernst in Göttingen. It was only after reading Lewis and Tolman's paper and possibly also Tolman's follow-up sometime in 1910 that Einstein would gradually drop talk of principles of relativity in favor of postulates, especially in the context of work in the new general theory.

The adoption of the American postulational terminology and perspective was consistent with Minkowski's own choice in 1908. His application of non-Euclidean geometry was not the sole distinctive disciplinary resource and incentive for mathematicians (and mathematical physicists). The axiomatic structure was part and parcel of the new mathematical approaches, at least in post-Euclidean geometry. To mathematicians it was as important to their reception, reconstruction and development of relativity as was the geometric perspective[37] and group algebraic invariants.

In fact, his choice of postulate terminology might seem somewhat perplexing, and stems from Minkowski's distinction between three different concepts of relativity: he had labeled the 'relativity theorem' Lorentz's ad hoc ('artificial') hypothesis, the 'relativity postulate', Poincaré's inductive generalization and Einstein's empirically motivated reconceptualization of time, and 'relativity principle' the generalized covariance principle for the relations between observable magnitudes (Corry, 2004: 193-4). The principle of relativity is, strictly speaking, the third of the axioms from which he derived the equations of electrodynamics. Following Hilbert's axiomatic epistemology in geometry, the goal was the clarification of relativity as an axiom probing the redundancy and inconsistency in the foundations of mechanics.

11. Robb's turn to axiomatic foundations of geometry beyond Russell

The contrast between Robb's 1911 and 1914 books parallels the contrast between Russell's 1903 and 1910-1913's books. It is not the exploration of logicism that had changed in Russell, but his turn to explicit axiomatic presentation in order to apply his philosophical method of analysis. He applied the logical techniques to problems in metaphysics, epistemology and philosophy of language. It is this method and the logical structure of axiomatic systems, not the logicist project, that Robb might felt prompted to follow.

262n.

[37] This is the focus of Walter (1999).

His exchange with Russell of 1908 might not have distanced him from Russell's general project in philosophy of mathematics as much as did his original physical perspective and the specific physical challenge posed by the theory of relativity. Robb and Russell pursued different philosophical and theoretical projects in relation to geometry. So Robb relied on alternative sources, Italian and American, older and more recent, along with some of their foundational attitudes and vocabulary.

Then in 1912 Robb made his acquaintance at least with some of the other protagonists of the axiomatics movement themselves, and his subsequent works would show it, citing them and not Russell –or even Hilbert. The special occasion was the International Congress of Mathematicians that took place in Cambridge in August 1912.[38] In the morning of August 23rd met a session of the Philosophy and History subsection of section IV (Philosophy, History and Didactics), chaired by Russell. The papers read addressed foundational issues with an emphasis on axiomatics: by G. Itelson (on the essence of mathematics), E. Zermelo (on the axiomatic and genetic methods), H. Blumberg (on a set of postulates for arithmetic and algebra), E.V. Huntington (on a set of postulates for abstract geometry) and A. Padoa (on the principle of induction) (Hobson & Love, 1913: vol 1, 53-4).

Peano too was attending the conference. Robb attended the session, likely among others, and participated alongside Padoa and N.A. Whitehead in the discussion of the paper by Blumberg, who had recently received his doctorate from Göttingen under Edmund Landau. Notice that it is the Continental-educated Americans, Blumberg and Huntington, who, as Robb would too, referred to sets of postulates. This linguistic marker helps tracks changing commitments to axiomatics and to the logical and epistemological status of axioms.

Veblen's preoccupation with the logical independence of axiom systems followed recent work by his advisor at the University of Chicago, E.H. Moore. Moore had criticized Hilbert's claim of the independence of his system of axioms by proving that one of them (II, 4) was redundant (Moore, 1902). Veblen also echoed Peano's defense of the primitive character of points and Padoa's recent criticism of Hilbert's set of basic terms, for basic elements, 'point', 'line', and 'plane', and for basic relations, 'situatedness' and 'betweenness,' are reducible to two undefined terms, 'point' and 'betweenness.'[39]

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[38] 22-18 August 1912, see Hobson and Love (1913), vol.1.

[39] Veblen (1903), Peano (1889) and (1894) and Padoa (1900).

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In the Italians' footsteps Veblen then proposed a system of axioms, or assumptions, and definitions for Euclidean geometry based on the undefined term 'point' for the basic class of elements and 'order' for the basic relation between them (Veblen, 1904; 1911). Veblen and Huntington picked up the theme of the independence of basic assumptions or postulates in their 1911 essays collected in a monograph edited by Young (Veblen, 1911; Huntington, 1911). It was also an issue Huntington addressed in his presentation at the Cambridge conference of 1912.

Shortly after, Robb turned his attention to Peano's system of axioms for the geometry of the straight line to examine their independence (Robb, 1913a). He concluded that one of the axioms, IX, could be proven, that is, it was not independent. Next in a follow-up note he published a second proof to the effect that also another axiom, VI, was, borrowing Veblen and Moore's terminology, reducible (Robb, 1913b). In 1914 he referred to Peano's axiom as redundant (Robb, 1914b: 105). In the aftermath of the Cambridge conference, the logical analysis of Peano's axioms provides the link to his system of geometric postulates that integrates the kinematic results of relativity theory.

The effects of the exposure and contribution to new developments in the foundations of geometry during the period 1911-1913 are evident in the transition between *Optical Geometry of Motion* (1911) and *A Theory of Time and Space* (1914). The distinctive aspect of the 1911 book is the emphasis on the physical interpretation; the distinctive aspect of the subsequent book of 1914 is the attention to the logical structure. In 1911 Robb didn't consider the specific set of geometric axioms and the logical structure it supported, nor their logical properties such as independence and consistency.

The focus on the foundations of geometry became the core of his approach to Einstein's resolution of the difficulties that beset electrodynamics in his theory of relativity. It informed his new view of the theory of relativity, developed more explicitly and in more detail in *A Theory of Time and Space* (1914). The book, however, should be understood in the tradition of Peano, Veblen and Huntington, with the particularity of the physical foundation that updates Helmholtz's project and integrates Einstein's results after Minkowski's geometric formulation. The subtitle of *Optical Theory of Motion*, *A New View of the Theory of Relativity* made clear the intended focus in 1911. Yet in 1914 the treatment of relativity is incidental except as the spatio-temporal framework for integrating the different strands in the foundations of geometry. The themes of formal logic, the elimination of paradoxes, the axiomatic approach to geometry, the physical interpretation of its axioms and the rejection of Poincaré's conventionalism (despite its endorsement of the empirical and mechanical bases for geometry) all appear mentioned in the Introduction to *Optical Geometry of Motion* (1911), but not developed. Robb defended a physical approach to the foundations of geometry, as an axiomatic system, rather than a logical approach in the logicist sense, that is, considering geometry a branch of formal logic (Robb, 1911: Preface). But it is in 1914 that he would recast the geometrical project in explicit terms of postulates with the inclusion of Veblen's and Huntington's meta-theoretical considerations of independence and consistency.

The central motivation was geometrical, the axiomatic foundations of geometry, and choice and interpretation of its axioms. Both are clearly set against a logical axiomatic structure of geometry. Robb sought explicitly to reject Poincaré's conventionalism (as well as Minkowski's abstract four-dimensional algebraic-geometrical treatment) and to adopt a position closer to Riemann's and Helmholtz's, distinguishing between axioms with a logical basis and axioms with a physical basis; the latter expressed the chronological ordering of mechanical and optical facts at spatio-temporal points. In the Preface to the book of 1911 Robb declared that geometry was from the standpoint of pure mathematics a branch of logic, but its origins lie with a physical problem (Robb, 1911: Preface). Another logical motivation concerned his articulation of relativity theory, as had done also for Einstein and Minkowski, namely, avoiding paradox or contradiction. Robb noted the second logical consideration in a footnote:

> It rather seems to the writer that the assumption of a unique time is intimately bound up with the logical principle of non-contradiction, whereby a thing cannot both be and not be at the same time. The conception of the index of a particle at an instant appears to avoid this difficulty. (Robb, 1911: 7n; Robb, 1914b: 2-4).

The emphasis on the physical, in connection to his early work on electron theory and radiation, dominated the treatment of geometry his title advertised as a new view of relativity theory. The role of logic, explicit but marginal in 1911, had become more prominent, even central, by 1914, in relation to the first explicit axiom system, presented in terms of postulates. There the same critical remarks about the relativity of simultaneity appear in the main text of the Preface (Robb, 1914b: 2).

The physical aspect, he noted, had been studied by Helmholtz and it could now supplement the logical aspects (Robb, 1914b: 1). From his teacher Minkowski's abstract physical geometry of space-time Robb adopted the

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graphic representation of the cone limiting subluminal velocities and the commitment to the integration of space and time. Minkowski had expressed the spirit of integration with these words: 'space by itself, and time by itself, are doomed to fade away into the shadows, and only a kind of union of the two will preserve an independent reality.' Robb expressed the integration and the absolute perspective with similar words, although from quote by Carlyle that begins as follows: 'But deepest of all illusory Appearances, for hiding Wonder, as for many other ends, are your two grand fundamental world-enveloping Appearances, SPACE and TIME.' (Robb, 1911: epigraph from Carlyle's *Sartor Resartus*, original emphasis). In 1914 Robb considered the axiomatic system of physical geometry to be expressing a logical theory of time that, he concluded, had to imply 'the Unchangeable.' The final words are also by Carlyle, and express the same fundamental role for time in the analysis of space and the consistent absolute perspective he had found in Minkowski rather than in Einstein: 'Know of a truth that only the Time-shadows have perished, or are perishable; that the real Being of whatever was, and whatever is, and whatever will be, is even now and forever.' (Robb, 1914b: 371). The emphasis on the absolute as an alternative to what he considered to be Einstein's logically paradoxical emphasis on relativity appeared in the title of the firs post-war version of the book, *The Absolute Relations of Space and Time* (1921). The focus would turn to an absolute time order.

With a focus on the (non-Euclidean) geometry of trajectories of particles and light flashes, geometry is then physical geometry, an optical geometry of motion. According to Robb, its basic axioms would include logical and physical propositions, the latter expressing optical rather than logical facts. In other words, optical facts constitute an interpretation of geometrical concepts (Robb, 1911: 2). But Robb offered no such axioms in *Optical Geometry of Motion* (1911). It was a matter of emphasis on the physical basis:

> It is proposed in the following pages to refer briefly, in the first place, upon which we might suppose some of the chief axioms to have their foundations.; and then to employ these to establish on a new basis some of the groundwork of the theory of "Relativity."
>
> The writer does not propose, in the present paper, to go into the more minute Logical details of the foundations of Geometry; as it seems to him that these would tend to obscure the general standpoint which he desires to emphasize. For this reason he prefers to reserve them for a future occasion. (Robb, 1911: 1-2).

In 1914 he still insisted on interpreting the theory of relativity as a particular approach to the more general 'investigation of the relations of Time and Space in connection with the physical phenomena of Optics.' (Robb, 1914b: 1). And he located the theoretical roots of the project, and of his contribution to it, in the electron theory of his teacher Larmor and of Lorentz.[40] According to Larmor, the electromagnetic properties of the electrons constituting ponderable matter could account for the null result of the Michelson-Morley experiment, the Lorentz-FitzGerald contraction hypothesis and the ensuing impossibility of distinguishing by optical or electromagnetic means between systems at rest or in uniform motion, which Robb referred to as Einstein's symmetries.[41]

In response to the problem of optics of moving bodies that dominated Cambridge and Göttingen physics and to Einstein's relativistic solution, Robb sought a particular physical basis for his system of geometry. It required absolute quantities and relations exhibiting an absolute order. With Helmholtz's precedent, he built on his own physical researches in the motion of radiating electrons. The geometry of coordinates describes the motion of elementary particles and flashes of light emitted between them, in Einstein's manner and recovering in passing Einstein's theory's kinematic results and, as a consequence, by putting it on a new basis (Robb, 1914b: 2).[42]

In 1911, for Robb the notion of congruence and the axiom that introduces it in the system of geometry receive physical meaning from the absolute character of the finite value of the speed of light. The physical interpretation involves the physical fact that light travels at a finite speed (but he did not assume isotropy, that the speed was the same in every direction). He considered the simultaneous emission of a flash of light to three different particles and the reception of the flash back at the emitting particle. Then he declared the lines spanning the distance between those particles as equal or congruent. Moreover, if flashes sent to two of the receiving and returning particles arrive simultaneously back at the emitting particle, the system of the three particles is not rotating in its plane (Robb, 1911: 4). He considers this physical scenario as giving physical meaning to the notion of absolute rotation.

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40 'Although generally associated with the names Einstein and Minkowski, the really essential physical considerations underlying the theories are due to Larmor and Lorentz.' (Robb, 1914b: 1).

41 'It was then shown by Larmor that the electromagnetic equations could, by a linear substitution, be made to assume the same form when taken with respect to a system moving with uniform velocity as they had when taken with respect to a system "at rest," and similar results were arrived at by Lorentz.' (Robb, 1914b: 1).

42 See also Windred (1933), Torretti (1983), Walter (1999).

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Finally he defined absolute distance in similar physical terms. He introduced a new variable associated with a particle at an instant, its index, which tracks the time order of the transmissions of light flashes between particles.[43] For Robb the focus on the index solved the psychological difficulty of non-unique time orders he associated with the relativity of simultaneity in the usual presentation of Einstein's theory, and the logical contradiction he perceived accordingly, with events simultaneously being and not being, at least from the absolute standpoint of a unique time (Robb, 1911: note). Simultaneity made sense only locally; and Robb sought to avoid identifying time determinations at distant positions. He distinguished an index of departure (emission), N_d, an index of arrival, N_a, and an index of return (reception), N_r (the first and last indices are properties of the first particle, the second index, a property of the second particle).

Absolute distance to a particle in motion relative to a fundamental particle in the so-called system of permanent (geometric) configuration is the same for each particle in the system and is defined by ½(N_r-N_d) and is always positive (echoing Minkowski's postulate). The index of arrival becomes ½(N*r*+N*d*), which may not coincide with the actual instant of arrival or require the same speed for the propagation of light in each direction (Robb, 1911: 7).

Since the representation of distance is physically based on the propagation of light, it is restricted to the scope of its reach. Then the uniform motion of a particle on a place relative to the index of a particle in the permanent system will be represented by a line at an angle smaller than 45 degrees; the latter is the angle of the line corresponding to the propagation of light. Robb calls this the standard cone with respect to a point on the index line (Robb, 1911: 8).

To conclude the optical interpretation the system of physical geometry, Robb ventured a physical interpretation of the particle's index. The most significant aspect is its reliance on his earlier research in the electronic theory of spectral lines:

> The number of vibrations corresponding to a definite spectrum line of a particular substance, which are executed in any interval, is proportional to the difference of index of the particle emitting the light at the beginning and end of the interval, the constant of proportion being fixed for each particular line. (Robb, 1911: 32).

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43 He acknowledged that a particle's index was effectively a measure of time, but only in the particle's neighborhood, but he dismissed the general identification of the two concepts (Robb, 1911: 5).

The physical facts at the basis of the space-time geometry yield a new geometry. He avoided conflict with classical intuitions, the Galilean law of additivity of velocities is preserved by a non-Euclidean, hyperbolic, triangle between the lines of three particles representing new quantities he called uniform relative "rapidities." It recalled Minkowski's geometrical treatment.[44] For a rapidity w, the absolute velocity relative to a non-rotating system of permanent configuration is v=tanh w.

Only in the more systematic explicit treatment of 1914 did Robb present an axiomatic system of geometry after the example set by Huntington and Veblen (and their Italian predecessors such as Peano and their system of geometric axioms) with their rejection (not Russell's or the Germans') of an exclusive talk of axioms (Veblen, 1911).[45] By then, as I pointed out, Robb had become acquainted with their work –and in 1912 two of the authors themselves– and followed in their footsteps with a critical examination of Peano's system. In his system of algebra, Huntington had introduced the fundamental concept of element and the relations of rule of combination; in geometry Peano and Veblen had adopted the fundamental concepts of points and the relations of spatial betweenness and order, respectively.[46] For the system of physical geometry based on a logic of time, Robb introduced the fundamental concept of elements and the relations of so-called conical ordered based on temporal relations of before and after. Robb was seeking to integrate what he considered new physical geometry, kinematics, and the new logical foundations of pure geometry. He was not alone; in 1912 Huntington and Robert Carmichael had formulated different postulate systems for relativity theory. Still, Robb cited neither despite his recent familiarity with at least Huntington's work and adoption of his terminology.[47]

Robb's system of geometry rests on twenty-one postulates and separate definitions. The kinematic structure of relativity becomes absorbed into a non-Euclidean, hyperbolic structure of physical geometry that integrates, after Minkowski's abstract geometry, both space and time, that is, spatial and

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44 Robb mentions a similar derivation by Sommerfeld from Minkowski's postulate of relativity (Sommerfeld, 1910a; 1910b). As Barrow-Green and Gray note, Varičak arrived independently at a similar result in 1912.

45 Barrow-Green and Gray focus on the post-war period and note Robb's references to Peano and Veblen in 1921 and then suggest the omission of Hilbert's name was due to a post-war anti-German sentiment; clearly the speculation doesn't apply to the same references in 1914.

46 Huntington (1902) and (1911), Peano (1894) and Veblen (1911); Veblen following Moritz Pasch rather than Hilbert.

47 Carmichael (1912) and (1913) and Huntington (1912); for details see Cat (2016).

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temporal orders. The axioms establish the so-called conical order out of two primitive concepts: *element* of a class or set and the relation of *after* between instants, or elements, of time. He concluded that he had 'shown how from some twenty-one postulates involving the ideas of *after* and *before* it is possible to set up a system of geometry in which any element may be represented by four coordinates *x*, *y*, *z*, *t*.' (Robb, 1914b: 369, original italics).

Robb insisted on the phenomenological nature of the system. The system of physical geometry is also a system of experienced geometry, '*a representation of the space and time of our experience in so far as their geometrical relations are concerned*.' (Robb, 1914b: 367). In fact, he placed the phenomenological dimension in the very determination of the fundamental concepts, that is, the elements of time standing in the fundamental relation are 'any two elements of time of which I am *directly* conscious.' (Robb, 1914b: 4). The very logical difficulty he had perceived behind the 'beauty and symmetry' at the heart of the relativity of simultaneity, 'that "a thing cannot both be and not be *at the same time*"' (Robb, 1914b: 2), he also cast in psychological terms, as 'something that was psychologically very strange.' (Robb, 1914b: 2). He was not alone; also Tolman and Lewis had cast the conceptual matter as a psychological difficulty (Lewis & Tolman, 1909; Tolman 1910). But this psychological dimension emphasizes the empirical dimension of physical space and time concepts more than it expresses any form of idealism. Robb doesn't explicitly defend any operationalist account of such concepts in terms of measurement procedures, as Einstein did, only their depedence on physical processes. As I show below, Robb also thought of logic, not just geometry, as a mechanical matter, in this case of a mental process that could be represented mechanically. In this sense, both his geometry and logic combine older mechanical foundations with the abstract logical features of the new axiomatics and foundations of mathematics.

The set of instants form a series in linear order satisfying five conditions (Robb, 1914b: 4, original italics):

(1) If an instant A be *after* an instant B, the instant B is not *after* the instant A, and is said the be *before* it.

(2) If A be any instant, I can conceive of an instant which is *after* A and also of one which is *before* A.

(3) If an instant A be *after* an instant B, I can conceive of an instant which is both *after* B and *before* A.

(4) If an instant B be *after* an instant A and an instant C be *after* the instant B, the instant C is *after* the instant A.

(5) If an instant A be neither *before* nor *after* an instant B, the instants A and B are identical.

Space-time points are the locations of physical events. In physical time those events satisfy what Robb called the *conical order*, represented geometrically by relations between sets of cones associated with any events at any space-time points. Thus, at any point occupied by a physical event we can define two cones sharing the same apex and axis, one opening upwards and another opening downwards. One event, B, is after another, A, if the point corresponding to B's space-time location lies within or on A's upward –that is, forward- cone. Similarly, B is before A if its space-time location lies within or on A's downward –that is, backward– cone. The order introduced by the cones is universal or uniform in the sense that at any different points cones share the orientation of their up-down axes and the cones' angle.

What makes the conical order a case of physical geometry is the physical meaning of the lines that define the cones. Robb distinguishes between three kinds of lines: inertial lines, optical lines and separation lines. *Inertial lines* are the lines connecting events inside a cone; they represent uniform motion. *Optical lines* are the lines defining a cone, making up its surface. They identify the physical meaning of the geometrical figure as the set of light rays leaving or arriving at a point at an absolute (universal) speed, or a two-dimensional light pulse. *Separation lines* are the lines connecting A to points outside its cones, that is, points that are neither before nor after A.

In physical space-time the conical order makes geometric sense of the kinematic fact that two events can be simultaneous only if they take place at the same spatial as well as temporal location (Robb, 1914b: 6, 13 and 47). The absolute character of the speed of light expresses the objective, absolute, significance of the inseparability of space and time, echoing Minkowski's insight. It's the same geometry that can express also Minkowski's hyperbolic structure of non-uniform relative motions, limited by asymptotes that represent optical lines, or the speed of light (Robb, 1914b: 364).

Robb developed the system of physical space-time geometry after the standard of axiomatics introduced in the systems by Peano, Veblen and Huntington. Half-way through Robb's exposition of his axiomatic system, theorem 69 relates the parallel optical lines running through three points connected by a separation line (Robb, 1914b: 100). Robb then draws an analogy with

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the notion of betweenness at the foundation of Peano's axioms of the straight line he had analyzed and criticized a year earlier (Robb, 1914b: 104; 1913a and 1913b). Next, after theorem 197, he argues that the three-dimensional geometry of the three types of lines for any three points are equivalent to the axioms of order from which Veblen deduced the system of Euclidean geometry (Robb, 1914b: 334).[48]

The role of Peano's and Veblen's systems, however, is more fundamental than Robb localized analogies suggest. His disavowal of axiom talk is a distinctive of Veblen, Young and Huntington and, although less consistently so, of Italians such as Peano and Padoa. And, as I mentioned above, talk of order can be traced to Veblen's assumptions of order in his monograph on the foundations of geometry, while talk of elements and postulates can be traced to Huntington's work, included in the same volume of monographs edited by Young containing Veblen's monograph (Robb, 1914b: 339; Huntington, 1911). Moreover, the first five postulates in Robb's system are analogous to some of the first assumptions or axioms in Veblen's and Peano's cited systems.

Robb's Postulate I of the Conical Order is 'if an element B be after an element A, then the element A is not after the element B.' (Robb, 1914b: 10). Veblen's corresponding assumption of order is Assumption II: 'if points A, B, C are in order [ABC] they are not in the order [BCA].' (Veblen, 1911: 5).

Robb's Postulate II is '(a) if A be any element, there is at least one element which is after A' and '(b) if A be any element there is at least one element which is before A.' (Robb, 1914b: 10). Peano's corresponding axiom is Axiom II: 'If A is any point, there is a point distinct from A.' (Robb, 1913a: 121; Robb, 1914b: 104).

Robb's Postulate III is 'if an element B be after an element A, and if an element C be after the element B, the element C is after the element A.' (Robb, 1914b: 10). The corresponding axiom in Peano's system is Axiom VIII: 'If A and D are distinct points, and C is a member of AD, and B of AC, then B is a member of AD.' (Robb, 1913a: 122; Robb, 1914b: 104).

Robb's Postulate IV is 'if an element B be after an element A, there is at least one element which is both after A and before B.' (Robb, 1914b: 10). The corresponding assumption in Veblen's system is Assumption IV: 'If A and B are two distinct points, there exists a point C such that A, B, and C are in the

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48 He cites Veblen (1911) and mentions the other references therein (Robb, 1914b: 339).

order [ABC].' (Veblen, 1911: 5). We can find a corresponding axiom also in Peano's system, Axiom IV: 'If A and B are distinct points, there is at least one point lying between A and B.' (Robb, 1913a: 121; Robb, 1914b: 104).

Robb's Postulate V is 'if A be any element, there is at least one other element distinct from A which is neither before nor after A.' (Robb, 1914b: 10). The corresponding assumption in Veblen's system is Assumption VI: 'There exist three distinct points, A, B, C, not in any of the orders [ABC], [BCA], [CAB].' (Veblen, 1911: 6).

Finally, Robb adopted from Veblen and Huntington also their concern with logical properties of the systems of postulates or assumptions such as consistency, sufficiency (or "categoricalness") and independence. They had addressed those properties in earlier work on systems of geometry and algebra,[49] and they get attention again in the monographs of 1911 (Veblen, 1911; Huntington, 1911).[50] At the end of his monograph Veblen actually deferred to Huntington's explanation (Veblen, 1911: 49). Huntington devoted a separate section to each property –followed by a defense of the use of the terms "postulate" and "assumption" instead of "axiom" (Huntington, 1911: 165-71).

Conditions or postulates are *consistent*, in Huntington's own terms, if there exists a system that satisfies them all (Huntington, 1911: 165). The basic postulates are *independent* if none is, as he put it, reducible or redundant, that is, none is a consequence of another (Huntington, 1911: 169). The system is *sufficient* if all the systems consisting of a class of elements and a basic relation or rule connecting them and satisfying the postulates are, in Huntington's terms, isomorphic, or can be put on a one-to-one correspondence (Huntington, 1911: 170).

Robb concluded the exposition of his system of geometry by turning briefly to the questions of consistency, independence and sufficiency. He had first noted in passing the sufficiency of the system of propositions as a basis for Euclidean geometry in the same way Veblen had defended his own (Veblen, 1904: 334 and 337). In the conclusion he raised the 'question of the consistency of the whole twenty-one postulates' and concluded that he had left 'little doubt that they are all consistent with one another.' (Robb, 1914b: 369-70). As I have mentioned, Robb had discussed the redundancy or independence of axioms first in 1913, criticizing the independence of one of Peano's axi-

[49] See for instance Huntington (1902) and Veblen (1904).

[50] Like Veblen and Huntington, also Robb might have been aware, while in Göttingen, of Hilbert's interest in the independence and consistency of axioms.

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oms in the footsteps of Veblen similar critique of Hilbert. He mentioned the result in relation to the analogy with Peano's system (Robb, 1914b: 105). In the conclusion he turned to his own system: 'The question as to whether the postulates are all independent is mainly a matter of logical nicety and is of comparatively little importance provided the number of redundant postulates be not large.' (Robb, 1914b: 370). He acknowledged that of the twenty-one postulates, not all are independent. Postulate II is a consequence of V and VI and that VI and XI could be combined (Robb, 1914b: 370).

In June of 1914, prior to the publication of the book, Robb wielded his logical approach in a letter to the editor of *Nature*. His target was Cunningham's recent papers on the principle of relativity. Robb questioned the conceptual consistency of Cunningham's formulation on the grounds that the idea of an absolute and definite velocity for light was inconsistent with the alleged indefiniteness (relativity) of measurements of length and simultaneity. He concluded with a veiled demand on Cunningham that expresses his commitment and the attempt to set the rules of the discussion of relativity theory accordingly: 'Query- What are Mr. Cunningham's fundamental concepts?' (Robb, 1914a: 454). Cunningham's reply must have seemed to Robb simplistic and logically inadequate, but it contained an acknowledgment of Robb's distinctive new perspective:

> the "fundamental" concepts in the representation of physical phenomena are space and time. But the articles did not profess to describe in detail a logical scheme of the universe of motion. Mr. Robb's forthcoming work in which this is attempted is anticipated with much interest. (Cunningham, 1914: 454)

12. The image and logic of the light cone: from intuitive methodological model to semantic logical model too

The transition of Robb's research interests from physics to physical geometry, that is, from electron theory to space-time theory, lays out a connected series of guiding elements. First, Einstein's theory was widely understood as a reformulation of electron theory; and electron research, especially in relation to radiation, connected the theoretical and experimental activities at Cambridge and Göttingen. Second, at Göttingen Minkowski shifted the focus on relativity from electrodynamics to space-time geometry. Third, geometry was the subject of sustained meta-mathematical, or foundational, analysis and reconstruction, with an emphasis on axiomatic systems and logical properties. I have discussed these elements above. In this final section, I discuss briefly a fourth connecting element: Robb's major works, in 1904, 1911 and 1914, feature conic geometric models with a varying meaning. The evolution is part of a longer genealogy preceding Robb's cones. By 1914, the geometrical model is not only a concrete physical model; it is also a logical model in the semantic, model-theoretic sense, at the service of logical claims. Moreover, Robb kept the term 'model' to designate both conceptions, the physical-geometric and the logical, providing continuity between both traditions.

1) *Geometric and physical models*

A geometric model is a geometric entity or a combination thereof with specific geometric properties satisfying the propositions of some abstract geometric theory such as Euclidean geometry. Its role is illustrative, to provide a particular example. A cone is a geometric model. Geometry was Robb's earliest scientific interest in Belfast. In the Cambridge tradition within which Robb would later study and write, a geometric illustration displayed spatial intuitive qualities that facilitated the application of more general and abstract relations such as mathematical equations. The cognitive value was of special use in communication or education situations such as problem-solving in examinations (Cat, 2001; Warwick, 2003). Alternatively, the concrete illustration might have methodological use. For instance, it provided the representation of geometrical properties of an empirical system for a predictive and explanatory purposes, as in astronomy or cartography, or constructive, as in architecture and engineering.

Geometric models are visualizable, but not always visual. Diagrams are the types of visual models representing geometric properties; they facilitate ultimately much of the communicative as well as computational and others roles. Three-dimensional objects with the relevant geometric features were typical educational aids. Robb was obviously familiar with geometric models and diagrams from his early interest in geometry and subsequent education at Cambridge.

Physical models were typically understood as specific representations of physical properties or relations according to more general and abstract representations. They had their own concrete instantiations, like geometric models, in this case three-dimensional concrete material objects. Whether descriptions or concrete material objects, physical models displayed selected relevant features. The selection was grounded on relations of instantiation and analogy. Even when the model described possible systems, the specific description was meant to satisfy the more abstract and general relations. This

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semantic relation of satisfaction enabled the functions of relative concreteness and analogy. Their different functions could be illustrative, computational, predictive or methodological, enabling the application of more general principles or relations to the representation of empirical systems.

Geometric models may be also part of physical models, populating treatments of optics and mechanics or research publications in those areas. Mechanical theorizing placed spatial properties at the center of the description of mechanical systems characterized by structural features such shapes and kinematic behavior such as trajectories. Diagrams provided adequate visual representations of the relevant spatial properties or analogs. While the traditional relation between geometry and mechanical and optical properties lied in practices and instruments of geometric construction and measurement, Robb subscribed to Riemann's and Helmholtz's foundational account of physical geometry provided a foundational relation, with principled accounts of the dependence of geometric notions and axioms on mechanical and optical phenomena. Physical models could be also part, then, of geometric ones.

Mechanical theory and 19th-century mechanical engineering placed mechanical models at the center of a diversity of scientific and technological practices. At Cambridge, the role of models was based on the fundamental cognitive status of spatio-temporal and mechanical features. The mathematical theory of mechanics, extended to the hypothetical ether, enabled the intuitive mathematization of phenomena such as the propagation of light and the action of electric and magnetic forces. In particular, Maxwell and Kelvin famously succeeded in bringing electromagnetic phenomena under the new mathematical theory of energy physics, including optics. Maxwell's legacy at the Cavendish and among followers beyond involved the appeal to mechanical models of fluid flows, elastic solids and connected spheres and cogwheels in education and electromagnetic research. Research in electron theory and radiation extended this tradition.

2) *Precursors to Robb's geometric and physical cones and their precursors*

Robb's use of conic models as geometric and physical models included two-dimensional diagrams. This tangle didn't just extend an earlier local modeling tradition. It also extended a more recent tangled genealogy of conic models both at Cambridge and Göttingen.

As I have mentioned above, Voigt derived a set of transformations (anticipating properties of Lorentz's) that preserved the velocity of light and applied this result to the propagation of wave surfaces in the shape of a light cone (Voigt, 1887: 48-50; Sommerfeld, 1910a: 666, Fig. 4). This is one in a series of cone models serving different but connected theoretical projects including Robb's. In several of Voigt's books, both on mechanics and on electricity and magnetism, conic models appear occasionally playing one of two roles: the illustrative application of a physical principle and the technique for solving a problem. In both cases the cone model operates as a geometrical constraint or boundary condition (Voigt, 1896; 1901).

In 1904 Robb followed Lorentz and Voigt in applying classical dynamics to explain Zeeman's sets of spectral lines in terms of the vibrating motion electrons. He postulated a structure of the radiating particle containing a coupled pair of electrons in oscillating in response to an elastic central force (Robb, 1904). Inspiration had come from a treatise on dynamics written by the Edward J. Routh, Cambridge's most successful Tripos examination coach, and which included examples from examination problems (Routh, 1892: vol. 2).[51]

As a possible geometric constraint on the coordinates and velocities of two bodies Robb borrowed from Routh's text the example of motion on a cone. The cone model serves in the treatise a tripe role of illustrating general physical principles, help with calculation (especially in the application of solid geometry to treatments of rotation) and help the student practice the formal application of the physical principles to specific geometric situations. A number of examples involving cones were borrowed from exam problems that Robb would have studied in preparation for his own Tripos examination.[52]

In the geometric treatment of rotation due mainly to Poinsot, Routh introduced a specific use of the model of the dynamics of a cone to represent time (Routh, 1892: vol. 2, art. 198):

> Poinsot has shown that the motion of the body may be constructed by a cone fixed in the body rolling on a plane which turns uniformly round the invariable line. If, as in the preceding theory, we suppose the plane rough, and to be turned by the cone as it rolls on the plane, the angle turned by the plane will measure the time elapsed.

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[51] See Robb (1904: 13-14).

[52] See, for instance, examples in arts. 198 and 229 of Routh (1892).

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Robb's strategy in his dissertation fits with the Cambridge tradition of 19th century mathematical physics, in which physical research was guided by the application of mathematics, especially the calculus, and the focus on certain systems as set in examination problems.[53]

V-shaped geometric space-time diagrams began appearing in electron theory after Robb defended his dissertation in February 1904. In July 1904 Sommerfeld presented before the Göttingen Royal Society of Sciences the two installments of his treatment of the electron's surface charge distribution at uniform speeds below and over the speed of light. He presented the second installment in June 1905. Both installments included space-time diagrams depicting electron motion (Fig. 1).[54]

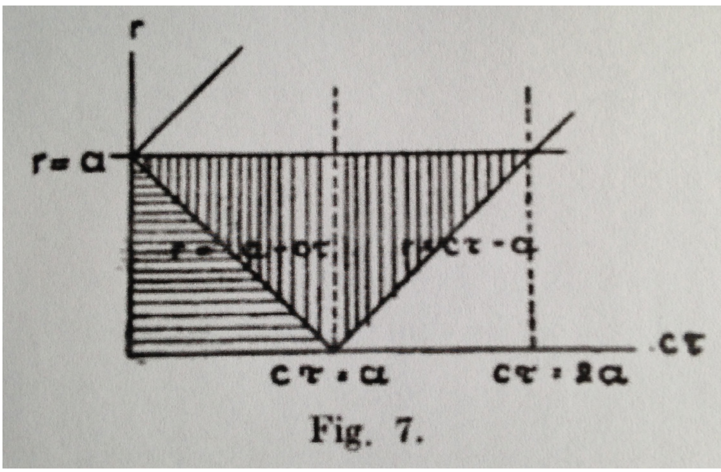

Figure 1. Sommerfeld's space-time diagram depicting electron motion (1904).

He was then professor of mechanics at Aachen, where graphic techniques were valued for use in engineering (Pyenson, 1979: 81).

One X-shaped diagram in Voigt's later *Magneto- und Elektrooptik* representing electric potential lines (Fig. 2) is graphically reminiscent of the space-time diagram in Sommerfeld's articles on electron theory of 1904 and 1905 (Voigt, 1908: 365, Fig. 71).

53 The role of the Cambridge Tripos exams in the introduction and research application of the calculus has been studied by Warwick in Warwick (2003). See also Cat (2001).

54 Sommerfeld (1904: 429, Fig. 7), and Sommerfeld (1905: 230, Fig. 14).

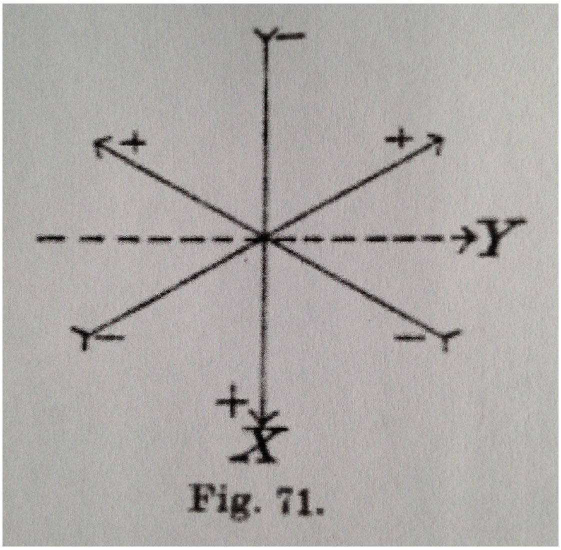

Figure 2. Voigt's spatial diagram for electric fields (1908).

Despite the graphic similarity with Voigt's diagram, it is Sommerfeld's explicit space-time diagrams that in turn constitute the immediate geometrical predecessor of the space-time cone diagram in Minkowski's 1908 lecture (Pyenson, 1979: 84). They all constitute a family of close graphic precursors of Sommerfeld's and Robb's subsequent cone diagrams, tools in their transition from electron theory to relativistic geometry in 1910 and 1911, respectively.[55]

Minkowski's use of space-time diagrams relied on a prior commitment to visual thinking, in particular through visual-geometrical intuition as a tool for discovery (Galison, 1979: 87). He had appealed to spatial and geometrical intuition first for insight into mathematical concepts in number theory in *Geometry of Numbers* (1896), then for insight into physics of space-time and electrodynamics (1908). Throughout he assumed a pre-established harmony between mathematics and nature.

For Minkowski relativity theory was justified as a contribution to the electromagnetic worldview; his contribution was clarifying how the motion of electrons was informed by the physical reality of space-time geometry.

[55] Sommerfeld discussed Minkowski's diagram especially in Sommerfeld (1910a: 752; 1910b: 666).

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Lorentz had constructed an electrodynamics of moving bodies on the back of mathematical transformations and hypothesis of physical contractions. Minkowski understood them from the standpoint of the aesthetic value of symmetry and the structure of groups of geometric rotations and four-dimensional invariants (an analysis borrowed from Poincaré). This algebraic interpretation received geometric expression in the representation of the invariant interval as a hyperboloid that contains the representation of all possible space-time coordinates of a physical event.

Minkowski represented graphically the hyperboloid on a V-like axis system two-dimensional space-time centered on an event placed at the origin of coordinates. The V-shape represented two-dimensionally a space-time cone. He then generalized the diagram to an X-shaped axis system that included the central event's past. The transformation curves are contained in what he called the fore-cone (*Vorkegel*) and the aft-cone (*Nachtkegel*) (see Figure 3) (Minkowski 1909/1911: vol. 2, 433 and 438).[56]

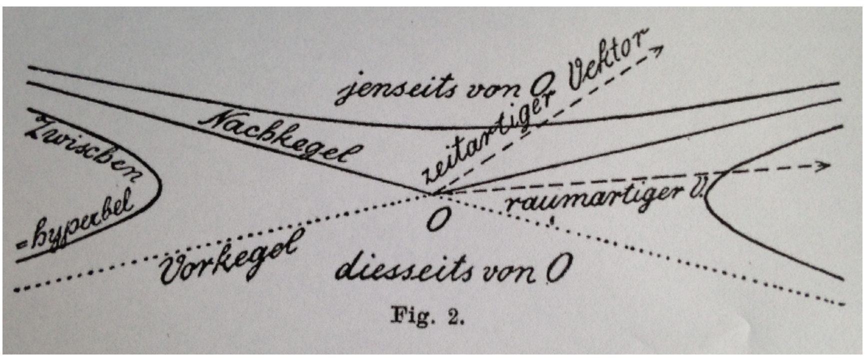

Figure 3. Minkowski's space-time diagram and light cones (1908).

The fore-cone contains all the (past) points that emit light towards the central event, while the aft-cone contains all the (future) points that receive light from it. They are light cones in the physical geometry of space-time.

[56] The cones are now known as the future and past cones, respectively.

3) *Robb's geometric & physical cone models.*

In *Optical Geometry of Motion* (1911) Robb introduced the geometry of uniform motions relative of a given particle –the fundamental particle– at a point on an axis that represented the particle's index. On this axis, the index represents local time in terms of the order of events of light emission and reception at the fundamental particle. The propagation of a flash of light emitted in the direction of a moving particle is represented by a line at a 45-degree angle relative to the index axis. Relative to the location C=(a, b, c) of the particle, all the lines representing the motion of light lie on a cone centered around the index axis, z, $(x-a)^2 + (y-b)^2 - (z-c)^2 = 0$. (Robb, 1911: 8). This is the physical light cone; he called it the *standard cone* relative to the point C. Robb extended the model relative to the location of a uniformly moving particle with two cones, representing reception and emission of light flashes, around its index line (Fig. 4) (Robb, 1911: 11). The geometric and physical properties are inseparable; the geometric model is in fact a model of physical geometry. Further, a diagram depicted the bi-conic geometric model with the geometric properties of the physical standard cone, that is, of a physical model.

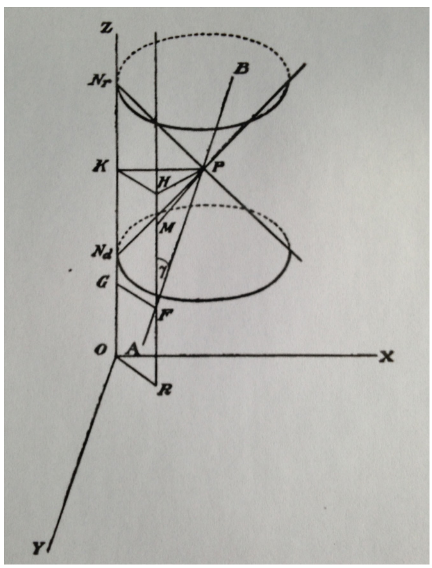

Figure 4. Robb's light cones (1911).

The cones allowed Robb the calculate relations between the indices of the particle in uniform relative motion and establish the non-Euclidean geometry for rapidities, which satisfy the classical relation of additive composition.

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Next Robb's geometric project underwent an "axiomatic" turn. As I have discussed above, in *A Theory of Time and Space* (1914) Robb represented the physical geometry of space-time embedded in a system of postulates. And to that effect, he re-introduced the conic model as a 'geometrical illustration.' (Robb, 1914b: 4). It illustrated the fundamental concept of order that he had borrowed from prior axiomatizations of Euclidean geometry. But the so-called *conic order* was a geometric model of the physical geometry of space-time and, like its 1908 and 1911 predecessors, it concerned local time relations of before and after between space-time events. The two relations required two cones; the geometric model is, again, a bi-conic model. Among the temporal relations, Robb was particularly interested in the relation of simultaneity, the source of much logical dissatisfaction towards Einstein's theory. The cone system illustrated the possibility of events that were neither before nor after each other. In his system, Robb restricted the relation of simultaneity to events occurring at the same place (Robb, 1914b: 6).

The model was also physical beyond the temporal dimension: the physical geometry of the cones was defined by limiting optical lines that represented the trajectory of light signals to or from a particle, that is, emission and reception events located at the cones' vertex. Lines representing the trajectory of particles at subluminal speeds are contained within the cones. The future and past cones contain the subluminal local physical histories accessible to the system at the vertex, the local conic order. As in 1911, Robb also portrayed the model graphically, with a geometrical diagram (Fig. 5) (Robb, 1914b: 5).

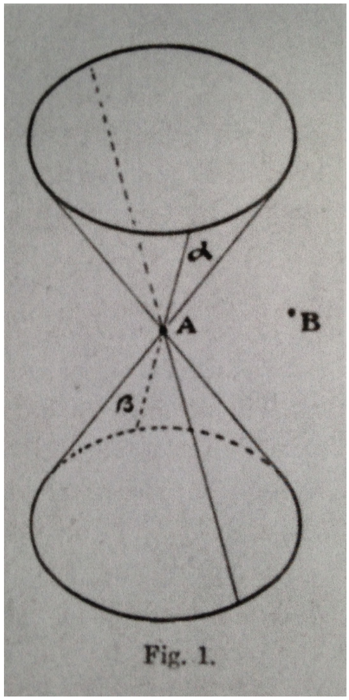

Figure 5. Robb's light cones (1914)

4) *Logical models and the logic of cones*

I have situated Robb's light cone model in a genealogy of graphic and physical cone models; it includes space-time diagrams physical models in electron theory and mechanics in his early work and work by teachers at Cambridge and Göttingen. Now, any elements of continuity get further complicated by the additional, logical role of the light cone model in 1914. The logical turn is the expression of Robb's engagement with recent trends in the foundations of mathematics that I have documented above, especially after two events: the publication of Young's volume of monographs in 1911, which included one by Veblen and another by Huntington, and the 1912 meeting of the International Congress of Mathematics at Cambridge with the presence of Peano, Padoa, Zermelo and Huntington –the last three speaking at the meeting of the Philosophy Section with Robb's attendance on record. Robb's new axiomatic standard for his space-time geometry included a concern with

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the (meta)logical dimensions of axiomatics that the Americans Veblen and Huntington introduced reaching beyond Russell's and Hilbert's work at Cambridge and Göttingen, respectively.

I call a logical model a semantic model or interpretation of symbolic axioms at the service of logical features of an axiom system. Interpretations of sentences make them truth-valued, they become models when they satisfy the sentences or make them truth in that interpretation. The modern understanding of model theory under this term is associated with Tarski, appearing early in his 1935 definition of logical consequence: 'The sentence X follows logically from the sentences of the class K if and only if every model of the class K is also a model [*Modell*] of the sentence X.' (Tarski, 1936/1956: 417).[57] Tarski was explicit that the concept of model reflected the semantic role in axiom theory:

> an arbitrary sequence of objects which satisfies every sentential function of the class L' [of sentential functions] will be called a *model* (*Modell*) or *realization* (*Realisierung*) *of the class L of sentences* (in just this sense one usually speaks of models of an axiom system of a deductive theory). (Tarski, 1936/1956: 417; Tarski, 1936/1986: vol. 2, 279, original italics).

In fact, von Neumann had already introduced the term 'model' (*Modell*) in his paper of 1925 with an axiomatization of set theory. His use adopted the semantic meaning that Veblen and Huntington had associated with sets, systems, classes or assemblages that satisfy a given system of axioms (or rather, in their case, assumptions or postulates): von Neumann presented as equivalent the statements 'to find a system Σ satisfying the axioms' ('*ein System Σ zu finden, welches den Axiomen genügt*') and 'find a model of set theory' ('*ein Modell für die Mengenlehre findet*') (von Neumann, 1925: 235).

Robb used the light cone model –that is, the bi-conic model– in the logical sense, as an interpretation of postulates relevant to proving properties of the system of geometry such as the independence and consistency of its postulates. In 1913 he had followed Veblen's and Moore's criticism of Hilbert's system of Euclidean geometry to criticize the independence of Peano's axioms. Now in 1914 he applied the same standard to prove the independence of the fifth condition of the conic order, about the possibility of events that are neither before nor after each other: 'that [the fifth condition] is in reality no logical consequence of the other conditions may be shown by the help of a geometrical illustration.' (Robb, 1914b: 4). He added: 'We may have two ele-

57 Original in Tarski (1936/1986: vol. 2, 278).

ments, of which one is neither before nor after the other, but which yet are not identical, without our being involved in any logical absurdity.' (Robb, 1914b: 6). By the end of the book, Robb seems to have loosened, at least temporarily, his commitment to the absolute value of independence: 'The question as to whether the postulates are all independent is mainly a matter of logical nicety and is of comparatively little importance provided that the number of redundant postulates be not large.' (Robb, 1914b: 370). What is significant is the acknowledgment and application of the new standards of axiomatics.

In the spirit of formalism, he also acknowledged that the status of the model is semantic, interpreting or illustrating the logically basic conditions without replacing them: the 'illustration is suggestive, but the development of our theory is in no logical sense dependent upon it.' (Robb, 1914, 4). The emphasis on illustration and interpretation provide the link between the Cambridge-style methodological type of model and the semantic one. Strictly speaking, as I have noted above, a semantic role is part of the Cambridge tradition of geometrical and mechanical models, but it is embedded in a theoretical framework, without the additional foundational, logical function I draw attention to by referring to logical models (Cat, 2001). The duality of types of models is clearly derivative from the duality of functions.

Another element of continuity between the traditions is terminological. The use of the same term 'model' in both contexts provides a linguistic link between the earlier, physical tradition and the more recent, semantic one in logic and axiomatics. Robb's use in the latter sense is one of the earliest instances of the use of the term with a semantic meaning in the new, model-theoretic sense. He had used it in his dissertation in German, *Modell*, in the mechanical and methodological senses familiar to German audiences mainly from the work of Maxwell and his British followers. The mechanical model of radiating electrons describes a connecting mechanism of interaction and the geometric properties of their constrained motion (here the cone is eventually introduced) (Robb, 1904: 65).[58] To the illustrating mechanical model, Robb added a visually illustrating graphic representation.

[58] The distinguishing features of the Cambridge, Maxwellian standard are evident in Robb's introduction of the model: 'Im Folgenden ist ein Modell beschrieben, das eine solche geometrische Verbindung illustriert und das vielleicht von einigem Interesse ist. Selbsverständlich beabsichtigen wir nicht damit zu sagen, daβ der Mechanismus dieser Modelle irgend etwas mit dem *wirklichen* Mechanismus, der die Elektronen verbindet, gemeinsam hat.' (Robb, 1904: 65, original italics).

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According to Robb, the model with the system of cones provided, an "interpretation", "visualization" and "clarification" of the fundamental relations of the conic order and other postulates:

> Results involving only three coordinates x, y and t may be visualized by means of the three-dimensional conical order described in the introduction, but a certain amount of distortion appears in a model of this kind, since equal lengths in the model do not in general represent equal lengths as we have defined them.
>
> The optical signification of the Posts. I to XVIII are however made clear by such models, and it is easily seen that the assertions made in these postulates, when interpreted in the manner described, are in accordance with the ordinarily accepted ideas.
>
> Post. XXI also finds an interpretation in such a model, but its significance is concerned rather with the logic of continuity than with any observable physical phenomenon. (Robb, 1904: 368-9)

Finally, Robb addressed the model's (meta)logical role in proving the consistency of the system of postulates:

> Of the postulates used: nineteen, namely I to XVIII and Post. XXI, may easily be seen to have an interpretation in three-dimensional geometry by making use of cones as described in the introduction.
>
> It follows that if ordinary geometry be consistent with itself, these nineteen postulates must be consistent with one another. (Robb, 1904: 369).

Robb's axiomatization of space-time geometry constitutes not only the earliest axiomatic treatment of non-Euclidean physical geometry as a presentation of relativity theory. It was also one of the earliest axiomatizations in physics by the new standards of axiomatics and with attention to its metamathematical or metalogical foundations.

In the succession of models of cones, I have identified dimensions of partial continuity: geometric models, mechanical models (motion of particles and light), graphic models (diagrams) and linguistic labels. In the tangled genealogy, Robb's shifting use of cones changed from the geometric and mechanical to including the graphic and the logical. In doing so, the evolution tracked his changing commitments and interests engaging related new developments in both physics –electron theory and relativity– and the foundations of mathematics –axiomatics and postulationism.

Conclusion

I have argued that Robb's transition to an axiomatic approach to both geometry and relativity is a synthesis of Cambridge electrodynamics, represented by Robb's mentors Joseph Larmor and J.J. Thomson, and modern foundations of geometry, especially axiomatics, represented by Russell in Britain and, as his references and use of postulates rather tan axioms indicate, also by Veblen, Young and Huntington in America. In addition, one may recognize in the synthesis the enabling role of familiar work in geometry, relativity and electrodynamics at Göttingen, especially by Hilbert, Sommerfeld, Minkowski and his dissertation advisor Voigt. The transition to work in space-time geometry and the role of new work in the foundations of mathematics may seem surprising that in the light of his neglected initial research. Here I have documented his subsequent engagement of new foundations of mathematics and argued that its role can be tracked by the evolving use and significance of cone models that I have documented in his different works. The role of foundational movement in Germany, Britain and America explains in turn the evolution of the cone models. It also places his research in seamless contact with physics, geometry, logic and the foundations of mathematics.

References

Barrow-Green, J. & Gray, J.J. (2006). 'Geometry at Cambridge, 1863-1940', *Historia Mathematica* 33, 315-356.

Campbell, N.R. (1910a). 'The principles of dynamics', *Philosophical Magazine* Ser. 6, 19, 168-81.

Campbell, N.R. (1911a). 'The common sense of relativity', *Philosophical Magazine* Ser. 6, 21, 502-517.

Campbell, N.R. (1919). *Physics. The Elements*, Cambridge: Cambridge University Press.

Carmichael, R.D. (1912). 'On the theory of relativity: Analysis of the postulates', *Physical Review* 35, n.3, 153-176.

Carmichael, R.D. (1913). *The Theory of Relativity*, New York: John Wiley & Sons.

Cat, J. (2001). 'On understanding: Maxwell on the methods of illustration and scientific metaphor', *Studies in the History and Philosophy of Modern Physics* 33B, n.3, 395-442.

ISSN 0719-4234 / eISSN 0719-4242

Cat, J. (2016). *Intellectual Trajectories and Local Interactions: Carmichael and Davis from Mathematics to Relativity and Philosophy at Indiana University in the 1910s and 20s*, Indiana University ms.

Corcoran, J. (1980). 'On definitional equivalence and related topics', *History and Philosophy of Logic* 1, 231-4.

Corry, L. (1997). 'Hermann Minkowski and the postulate of relativity', *Archive for History of Exact Sciences* 51, n.4, 273-314.

Corry, L. (2004). *David Hilbert and the Axiomatization of Physics (1898-1918)*, Dordrecht: Kluwer.

Cunningham, E.C. (1914). 'Letter to the editor: The Principle of Relativity', *Nature* 92, 2331, 454.

Darrigol, O. (2000). *Electrodynamics from Ampère to Einstein*, Chicago: University of Chicago Press.

Darrigol, O. (2014). *Physics and Necessity*, Oxford: Oxford University Press.

DiSalle, R. (1993). 'Helmholtz's empiricist philosophy of mathematics: Between laws of perception and laws of nature', in D. Cahan, ed., *Hermann von Helmholtz and the Foundations of Nineteenth-Century Science*, Berkeley: University of California Press, 498-521.

Ferreirós, J. (2006). 'Riemann's *Habilitationsvortrag* at the Crossroads at Mathematics, Physics and Philosophy', in J. Ferreirós and J.J. Gray, eds., *The Architecture of Modern Mathematics: Essays in History and Philosophy*, Oxford: Oxford University Press, 67-96.

Galison, P.L. (1979). 'Minkowski's space-time: from visual thinking to the absolute world', *Historical Studies in the Physical Sciences* 10, 85-121.

Goldberg, S. (1984). *Understanding Relativity: Origin and Impact of a Scientific Revolution*, Boston: Birkhäuser.

Hilbert, D. (1899). *Grundlagen der Geometrie*, Leipzig: Teubner.

Hobson, E.W. & Love, A.E.H. (eds.) (1913). *Proceedings of the Fifth International Congress of Mathematicians*, 2 vols., Cambridge: Cambridge University Press.

Hunt, B.J. (1991). *The Maxwellians*, Ithaca, NY: Cornell University Press.

HUNTINGTON, E.V. (1902). 'A complete set of postulates for the theory of absolute continuous magnitude', *Transactions of the American Mathematical Society* 3, n.2, 264-79.

HUNTINGTON, E.V. (1911). 'The fundamental propositions of algebra', in J.W.A. Young, ed., *Monographs on the Topics of Modern Mathematics*, New York: Longmans, Green and Co., 151-210.

HUNTINGTON, E.V. (1912). 'A new approach to the theory of relativity', *Philosophical Magazine* 23, n.136, 494-513.

KOX, A.J. (1997). 'The discovery of the electron: II. The Zeeman effect', *European Journal of Physics* 18, 139-44.

LARMOR, J. (1900). *Aether and Matter*, Cambridge: Cambridge University Press.

LAUB, J. (1910). 'Über die experimentellen Grundlagen des Relativitätsprinzips', *Jahrbuch der Radioaktivität und Elektronik* 7, 405-463.

VON LAUE, M. (1911). *Das Relativitätsprinzip*, Braunschweig: Vieweg.

LENNES, N.J. (1911). 'Review: Edward V. Huntington, *The Fundamental Laws of Addition and Multiplication in Elementary Algebra*', *Bulletin of the American Mathematical Society* 17, n.7, 362-5.

LEWIS, G.N. & TOLMAN, R.C. (1909). 'The Principle of Relativity, and non-Newtonian mechanics', *Proceedings of the American Academy of Arts and Sciences* 44, n. 25, 711-24.

LORENTZ, H.A. (1916). *The Theory of Electrons and Its Applications to the Phenomena of Light and Radiant Heat*, Leipzig: B.G. Teubner.

MANCOSU, P. (2003). 'The Russellian influence on Hilbert and his school', *Synthese* 137, 59-101.

MILLER, A.I. (1981). *Albert Einstein's Special Theory of Relativit: emergence (1905) and Early Interpretation (1905-1911)*, New York: Springer.

MINKOWSKI, H. (1909/1911). 'Raum und Zeit', *Physikalische Zeitschrift* 10, 104-111, in *Gesammelte Abhandlungen*, ed. D. Hilbert, 1911, vol. 2, 431-444.

MINKOWSKI, H. (1915). 'Das Relativitätsprinzip', *Annalen der Physik* 47, 927-38.

MOORE, E.H. (1902). 'On the projective axioms of geometry', *Transactions of the American Mathematical Society* 3, 142-158.

ISSN 0719-4234 / eISSN 0719-4242

von Neumann, J. (1925). 'Eine Axiomatisierung der Mengenlehre', *Journal für die reine und angewandte Mathematik* 22, 219-40.

Padoa, A. (1904). 'Un nuovo sistema di definizioni par la geometria euclidea', *Periodico di Matematische* 1, 74-80.

Peano, G. (1889). *I Principii di Geometria Logicamente Esposti*, Torino: Fratelli Bocca Editori.

Peano, G. (1894). 'Sui fondamenti della geometria', *Rivista di Matematica* 4, 51-90.

Poincaré, H. (1902). *La Science et l'Hypothèse*, Paris: Flammarion.

Pressland, A.J. (1892). 'On the history and degree of certain geometrical approximation', *Proceedings of the Edinburgh Mathematical Society* 10, 27.

Pyenson, L. (1976). 'Einstein's early scientific collaboration', *Historical Studies in the Physical Sciences* 7, 83-123.

Pyenson, L. (1979). 'Physics in the shadow of mathematics: The Göttingen electron-theory seminar of 1905', *Archive for History of the Exact Sciences* 21, n.1, 55-89.

Robb, A.A. (1904). *Beiträge zur Theorie des Zeemaneffektes*, Leipzig: Johann Ambrosius Barth.

Robb, A.A. (1905a). 'On the conduction of electricity through gases between parallel plates. Part I', *Philosophical Magazine* Ser. 6, 10, n.56, 237-42.

Robb, A.A. (1905b). 'On the conduction of electricity through gases between parallel plates. Part II', *Philosophical Magazine* Ser. 6, 10, n.60, 664-76.

Robb, A.A. (1906). 'The revolution of the corpuscle', *Nature* 1892, n. 73, 321.

Robb, A.A. (1911). *Optical Geometry of Motion. A New View of the Theory of Relativity*, Cambridge: W. Heffer and Sons.

Robb, A.A. (1913a). 'Proof of one of Peano's axioms of the straight line', *The Messenger of Mathematics* 42, 121-3.

Robb, A.A. (1913b). 'Note on the proof of one of Peano's axioms of the straight line', *The Messenger of Mathematics* 42, 134.

ROBB, A.A. (1914a). 'Letter to the editor: The Principle of Relativity', *Nature* 92, 2331, 454.

ROBB, A.A. (1914b). *A Theory of Time and Space*, Cambridge: Cambridge University Press.

ROBB, A.A. (1921). *The Absolute Relations of Time and Space*, Cambridge: Cambridge University Press.

ROUTH, E.J. (1892). *An Advanced Treatise on the Dynamics of Rigid Bodies*, 2 vols., Cambridge: Cambridge University Press.

RUSSELL, B. (1899). 'Sur les axiomes de la géométrie', *Revue de Métaphysique et de Morale* 7, 684-707.

RUSSELL, B. (1903). *Principles of Mathematics*, Cambridge: Cambridge University Press.

RUSSELL, B. (1905a). 'Review of Henri Poincaré's *Science and Hypothesis*', *Mind* 14, 412-418.

RUSSELL, B., (1905b). 'On Denoting', *Mind* 56, 479-493.

RUSSELL, B. (1914). 'The relation of sense-data to physics', *Scientia* 16, 1-27.

SÁNCHEZ-RON, J.M. (1987). 'The reception of Special Relativity in Great Britain', in T. Glick, ed., *The Comparative Reception of Relativity*, Dordrecht: Reidel, 27-58.

SCANLAN, M. (1991). 'Who were the American postulate theorists?', *Journal of Symbolic Logic* 56, n.3, 981-1002.

SCHLIMM, D. (2003). 'Axioms in mathematical practice', *Philosophia Mathematica* (III) 21, 37-92.

SOMMERFELD, A. (1904). 'Zur elektronentheorie. II. Grundlagen für eine allgemeine Dynamik des Elektrons', *Nachrichten v.d. Königlichen Gesellschaft der Wissenschaften z. Göttingen* 1904, n.1, 363-439.

SOMMERFELD, A. (1905). Zur Elektronentheorie. III. Ueber Lichtgeschwindigkeits- und Ueberlichtgeschwindigkeits-Elektronen', *Nachrichten v.d. Königlichen Gesellschaft der Wissenschaften z. Göttingen* 1905, n.3, 201-235.

SOMMERFELD, A. (1910a). 'Zur Relativitätstheorie. I. Vierdimensionale Vektoralgebra', *Annalen der Physik* 337, n. 9, 749-76.

SOMMERFELD, A. (1910b). 'Zur Relativitätstheorie. I. Vierdimensionale Vektoranalysis', *Annalen der Physik* 338, n.14, 649-689.

Tarski, A. (1936/1956). ‘On the concept of logical consequence’, *Logic, Semantics, Metamathematics*, Oxford: Clarendon Press, 409-20.

Tarski, A. (1936/1986). ‘Über den Begriff der logischen Folgerung’, in S.R. Givant and R.N. McKenzie, eds., *Alfred Tarski. Collected Papers*, vol. 2, 1935-1944, Boston: Birkhäuser, 269-82.

Thomson, J.J. (ed.) (1910). *A History of the Cavendish Laboratory 1870-1910*, Cambridge: Cambridge University Press.

Tolman, R.C. (1910). ‘The second postulate of relativity’, *Physical Review* Ser. 1, 31, n.1, 26-40.

Torretti, R. (1978). *Philosophy of Geometry from Riemann to Poincaré*, Dordrecht: Reidel.

Torretti, R. (1983/1996). *Relativity and Geometry*, New York: Dover.

Veblen, O. (1903). ‘Hilbert’s foundations of geometry’, *The Monist* 13, n.2, 303-9.

Veblen, O. (1904). ‘A system of axioms for geometry’, *Transactions of the American Mathematical Society* 5, n.3, 343-384.

Veblen, O. (1911). ‘The foundations of geometry’, in J.W.A. Young, ed., *Monographs on the Topics of Modern Mathematics*, New York: Longmans, Green and Co., 3-54.

Veblen, O. & Young, J.W.A. (1910). *Projective Geometry*, 2 vols., Boston: Ginn and Co.

Voigt, W. (1887). ‘Ueber das Doppler’sche Princip,’ *Nachrichten v.d. Königlichen Gesellschaft der Wissenschaften u.d. Georg-August-Universität z. Göttingen* 1887, n.2, 41-51.

Voigt, W. (1896). *Kompendium der theoretische Physik*, 2 vols., Leipzig: Verlag von Veit & Comp.

Voigt, W. (1899). ‘Weiters zur Theorie der Zeemaneffectes’, *Annalen der Physik* 68, 352-364.

Voigt, W. (1901). *Elementare Mechanik*, Leipzig: Verlag von Veit & Comp.

Voigt, W. (1908). *Magneto- und Elektrooptik* Leipzig: B.G. Teubner.

Walter, S. (1999). ‘The non-Euclidean style of Minkowskian relativity’, in J.J. Gray, ed., 1999, *The Symbolic Universe*, Oxford: Oxford University Press, 99-127.

Warwick, A.C. (2003). *Masters of Theory: Cambridge and the Rise of Mathematical Physics*, Chicago: University of Chicago Press.

Whitehead, A.N. (1906). *The Axioms of Projective Geometry*, Cambridge: Cambridge University Press.

Whitehead, A.N. (1907). *The Axioms of Descriptive Geometry*, Cambridge: Cambridge University Press.

Whitehead, A.N. & Russell, B. (1910-1913). *Principia Mathematica*, 3 vols., Cambridge: Cambridge University Press.

Windred, G. (1933). 'The history of mathematical time: II', *Isis* 20, n.1, 192-219.