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A Higher-order Dialogical Logic to demonstrate Leibniz's Law of Identity of Indiscernibles *

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Abstract

In this note I discuss some issues around the law of Identity of Indiscernibles and, above all, its difference with the so-called law of indiscernibilty of identicals. In this way I distinguish between the notions identity, sameness and equality, through a phenomenological discussion and using the key idea of intentionality. In order to formulate the Leibnizian law of Identity of Indiscernibles, and examine its validity, we need higher order logic. I will give semantic rules for a second-order logic with identity in the framework of the dialogical logic, introduced by P. Lorenzen. Then I will demonstrate the validity of the law of Identity of Indiscernibles by means of the introduced logic.

Keywords: Identity of Indiscernibles, Dialogical Logic, Identity, Equality.

Resumen

Una lógica dialógica de orden superior para demostrar la ley de la identidad de los indiscernibles de Leibniz

En este trabajo discuto algunas cuestiones en torno a la ley de la identidad de los indiscernibles, especialmente respecto a su diferencia con la llamada ley de la indiscernibilidad de los idénticos. De esta manera, distingo entre las nociones de identidad, similaridad e igualdad a través de una discusión fenomenológica y utilizando la idea clave de la intencionalidad. Para formular esta ley leibniziana y para examinar su validez, necesitaremos de una lógica de orden superior. Daré entonces reglas semánticas para una lógica de segundo

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orden con identidad en el marco de la lógica dialógica, que fue introducida por P. Lorenzen. Demostraré así la validez de la ley de identidad de los indiscernibles por medio de la lógica introducida.

Palabras clave: identidad de los indiscernibles, lógica dialógica, identidad, igualdad.

Introduction

The principle of Identity of Indiscernibles, introduced by Leibniz, is of great significance in his metaphysics. It has intimate connections with the principle of sufficient reason and also with the fundamental principle of identity (which is the positive and more basic form of the law of noncontradiction). These principles altogether present an overall picture of Leibniz' monadological ontology. It is important to notice that these principles are not metaphysically trivial and they should not be considered as analytic, and contentless, truths, although once accepted they *maybe* considered as analytic in respect with the concepts which have accordingly modified. However, it is possible to think about the opposite cases and see what differences these principles make.

One unfortunate issue around the principle of identity of indiscernibles is that it is usually taken as closely connected to the indiscernibilty of identicals, and people often consider both together as the Leibniz' law. While these latter has no significant metaphysical indication and even, if taken unconditionally true, is simply wrong or at least not compatible with Leibniz' philosophy.

Both of the mentioned alleged laws rang over not only individuals but also predicates. Therefore, in order to speak about them precisely and progress in arguing for or against one of them in a logically precise manner, it is a good idea to formulate them in a higher order logic. So, this is the formulation of the principle of identity of indiscernibles (PII hereafter):

$$\forall x \forall y (\forall F (Fx \leftrightarrow Fy) \rightarrow x = y) \tag{1}$$

In this note I am going to employ a dialogical logic to provide a semantics which is able to demonstrate the validity of PII. Such a semantics would make us able to see the connections between the metaphysical law at work and the possible logical rules. Once we can formulate certain rules appropriate to reflect the metaphysical principle, we will be also able to analyze the principle from logical point of view and compare it with its rivals (or false associates).

I will first mention the ontological implications of the PII and that why it has intimate connections with the principle of sufficient reason and the principle of noncontradiction, without being possible to be reduced to them. Here I argue to distinguish between the notions *identity*, *sameness* and *equality*. Accordingly, I will explain why the alleged law of the indiscernibilty of identicals (II* hereafter), has no metaphysical indication, and if so taken it is either trivial or false.

However, the main aim of this writing is to introduce a higher order dialogical logic which is able to deal with PII and II*—and show the validity of the former while rejecting the latter. This would be based on the standard dialogical semantics introduced by Paul Lorenzen and then developed by Kuno Lorenz, Shahid Rahman and others. I will introduce a structural rule for indentity and two particle rules for quantifying over predicates. Then arguing for the intuitively aptness of these rules, I will apply the new obtained semantics to analyze the PII and II*. This is in fact a part of a larger project to show that how dialogical method is very apt to deal with metaphysical issues and at least it causes no harm like those raised within some other more common semantical frameworks. In the current work I will of course focus to discuss this point in respect to the Leibnizian metaphysics and the issue of identity there and the significance of the PII.

1 Identity, Sameness, Equality

One of the controversial issues of logics concerning metaphysics, such as first order modal logic, is the notion of identity. What does it mean to consider two things identical? Is this really a relation? Can it be discovered or it is just a matter of definition? To think in either of these ways has its own supporters and of course some challenges. However, the acquaintance with the answer is just presupposed in PII. So, first of all we should make the conception at work clear. In order to just declare that what Leibnizian account of identity used in the PII is, I will briefly survey the possible meanings of the term "identity.[a]nd try to make the possible distinctions explicit.

My theses is to distinguish between three ways in which we apply the concept identity. The method I have used to reach this point is investigation on intentionality as introduced by transcendental phenomenology. [1] However I do not aim to go to the details here and I just take it for granted that phenomenology is completely compatible with monadology as also explicitly declared by Husserl in various occasions. According to phenomenology intentionality is the main feature of consciousness and

1. For an explanation of the status of intentionality see (Husserl, 1982), and particularly for the role of intention in the constitution of objectivities see (Husserl, 1973).

every act of cognition, including the case of identity, should be analyzed on the basis of the intentions at work. Having this point in mind we depart to consider what is the meaning (or meanings) of identity.

A first notion is that notion of identity we use to indicate an ontological unity, in the sense that every existing entity has a peculiar identity. Identity here can be considered, in a loose manner, as a relation between a thing and itself (and nothing else). Here we just presupposed the predetermination of the existing entity. It can be a monad whose existence can not be analyzed, or another entity genuinely grounded on the monads. In fact this is the core meaning of identity. Nevertheless we have two other notions which are very close to this and sometimes are represented by the relation of identity and the symbol =.

If we consider a more common case in which the thingness of an entity is determined by acts of us (the egos) then it would be pointless to restrict the notion of identity to the ontological one. In the constitution of thingness, we first deal with essence rather than existence, then some parts of being are recognized as instances of that essence (or tokens of a type). So, we consider two things as same when they each fulfill a different intention while both are fulfilling a same *objectifying* intention which gives them their thingness. Therefore, as an object their identity is same but they are distinguished since recalled by two different, not objectifying, intentions, Consider this example: Alice is suffered from the same illness that I had last year. Here my illness and Alice's are same. First we had two phenomena conceived by two different intentions, one of those is an abnormal state in Alice's health situations and the other is an abnormal state in my previous health situation. However, so far we have no identity, namely such an abnormality need not be considered as a unity, it could be just a moment of another phenomenon. But at the same time we recognize that it fulfills an intention toward constituting a particular illness, and moreover they both fulfill this latter, so that in regard to their identity, namely what makes them ontologically distinguished, they are the same. Here we have a genuine relation. So, in this case, perhaps not in the previous one, we first conceive identity as a relation, a relation between two beings, each constituted by different, and not essence-constituting, intentions, that fall under a same kind, i.e. their thingness is due to a same objectifying intention. I will call this notion *sameness*.

Beside the sameness we have another relational notion very close to identity which is called *equivalence*. In equivalence we have two different objectifying intentions that their fulfillments are concurrent. Therefore, in contrast to sameness in which we have one objectifying intention with two not co-fulfilled, different intentions. Here we have two co-fulfilled (or to-be-fulfilled) objectifying intentions—and of course a third rang of intentions which are at work to recognize this co-fulfilling.

When we mean a same thing by means of two different ways in a manner that it is recognized, or just posited, that these two ways always signify a same thing, we have a case of equivalence. As for examples, consider the cases of the morning star and the evening star, or water and H_2O, and so on. Notice that here it is not the case that we have two signs to refer to a same object rather two meanings, two intentions, that happen to signify a same object. Basically we can have equivalence in two cases: 1- definition and 2- recognition of the sameness. This latter can be based on the existence of the referred object, like in the case of the morning star and the evening star, or just on the determining essences, like in the case of Zeus and Jupiter.

I introduce the following notations to indicate these different notions:

$=_i$ Ontological identity

$=_s$ Sameness

$=_e$ Equivalence

We have:

$x =_i y \rightarrow x =_s y$

$x =_i y \rightarrow x =_e y$

$x =_e y \rightarrow x =_s y$

It says that an identical thing has the relation of sameness with itself namely it falls under any kind that it falls (or it is a token of any type of which it is a token). Also an identical thing is equivalent with itself namely it is the same object though objectified differently. The third case says that if two objectifying intentions meet each other in an object there is an objectifying intention to be fulfilled by this very object. This is obvious because this latter intention can be one of the formers. For example 12 and 5+7 are equivalent; accordingly they are a same number. Assume that a number is constituted as a successor of an already constituted number, so there is an objectifying intention (here a number-constituting act) which is fulfilled by both; that is, both 12 and 5+7 are recognized as 11+1 and both are considered as a same object.

Now if we interpret PII as concerning ontological identity, namely if we have:

$$\forall x \forall y (\forall F (Fx \leftrightarrow Fy) \rightarrow x =_i y) \tag{2}$$

Then we would also have:

$$\forall x \forall y (\forall F (Fx \leftrightarrow Fy) \rightarrow x =_s y) \tag{3}$$

and:

$$\forall x \forall y (\forall F (Fx \leftrightarrow Fy) \rightarrow x =_e y) \tag{4}$$

Therefore, if in the case of PII, the notion of identity at use is ontological identity, as I am going to argue for it, there would be no problem if we leave the indexes of the symbol =. But in the case of II* it makes a lot of difference and if one defends the validity of II*, one should be explicit about which meaning of identity one is talking.

From the above formula the validity of 3 and 4 is already obvious; indeed they are not metaphysical claim but just true due to the meaning of the terms. For two things if it holds that each one of them possesses the predicate that the other does then it also possesses the essential predicates so that it should be considered as being of a same essence, as being the same object, as the other. Also if two things fall under the same descriptions, so that they always fulfill same intentions, then they would fulfill concurrently any objectifying intention too, thus they are equivalent.

This is the formula 2 which claims a metaphysically significant principle. If we interpret PII, as merely what 3 or 4 are saying, then Leibniz has not said something important, something that makes a difference in our view on ontology, and it can hardly be called a principle. In the next section I am going to mention some of metaphysical indications of the PII.

On the other hand, let us think about the II*. Considering the different notions of identity we have these three claims:

$$\forall x \forall y (x =_i y \rightarrow \forall F (Fx \leftrightarrow Fy)) \tag{5}$$

$$\forall x \forall y (x =_s y \rightarrow \forall F (Fx \leftrightarrow Fy)) \tag{6}$$

$$\forall x \forall y (x =_e y \rightarrow \forall F (Fx \leftrightarrow Fy)) \tag{7}$$

The first one, the formula 5, is trivially true. It says that any identical object falls under any predicate that it falls. No big deal. However the other ones are just false. For the sameness it is clear; two things that are same from one aspect of course can be different from other aspects. For the equivalence, although the two things are co-extension, the intensional features can be different, and from both phenomenological and monadological point of view intensional features are able to be genuinely objective features. This goes well in the case of recognition of the sameness: the morning star has something which makes it different from the evening star and it is a contingent truth that they happen to be a same planet. It is also true in the case of definition: in any case the defined goes (or may go) beyond the definer; otherwise it would be pointless, it would be just a matter of sign not a genuine equivalence relation.[2] For example let's define “$m * v^2$.ªs “energy”. Although it introduces us for the first time to the notion of energy in natural world, it is possible to investigate further relations and, from now on, to say that energy is so and so is not equal to say that “$m * v^2$ïs so and so. To assign two different signs to a same reference is not definition and in such a case we have no objective equivalence relation—unless we explicitly speak about the signs as objects.

Therefore, II* in one reading is trivial and in the others is false. That is why I think that it is unfortunate to attribute this claim to Leibniz. The problems that arise from taking for granted the formula 7 have nothing to do with Leibniz's philosophy. And to say a contentless truth like 5 as a metaphysical principle is far from Leibniz's so challenging philosophical attitude.

It is very important to notice the difference between the formula 7 and the principle of substitution which is widely used in reasoning and specially in computation: once we have that a=b we may substitute b in lieu of a in any step of reasoning. The principle of substitution is used in respect to a given context and it is not supposed to concern all aspects of the objects under question. It is beyond the scope of this not to study the issues around the principle of substitution, however I hope once the genuine meaning of PII will be clarified, it will help to remove some confusions between these two principles.

2 The status of discernibility

Now it is highly important to notice that Leibniz's PII is about ontological identity. If we read Leibniz carefully it will be clear that he just presupposes that it is indubitable that it is absurd for two things to be ontologically identical, and indeed the PII is a kinds of *reductio ad absurdum*. The principle, if accepted as true, would say that since it is absurd that two things be ontologically identical then there should be at least a property belonging to one of them and not the other so that distinguishes the two entities. Such a principle not only is not restricted to equivalence or sameness but also serve as a ground to explain that how it is possible to speak about these latter notions. In equivalence and sameness we have some objectifying intentions at work, as explained above. In the terminology of the era of Leibniz, they refer to such kind of intentions under the term “principle of individuation”. We will see in the following that how Leibniz shows that the principle of identity of indiscernibles is necessary in order the principle of individuation to

2. For an explanatory analysis of the case of definition and that the defined and the definer are not unconditionally interchangeable see (Husserl, 2002, pp. 231-234).

work. What Leibniz says is that just like it is absurd for two entities to be one it is also absurd that two entities be absolutely same in any respect; thus he rejects both atomism and the existence of a completely homogeneous being.

In the paragraph 9 of the *monadology*, Leibniz says:

> There are never two things in nature which are perfectly alike and in which it is impossible to find a difference that is internal or founded on an intrinsic denomination. (Leibniz, 1989, p. 643)

One may think that he exclusively speaks about monads, but this is not the case, for we should notice that:

> ...compound things are in symbolic agreement with the simple. (Leibniz, 1989, p. 649)

In general, for Leibniz, the well founded phenomena reflect essential features of monads, except for being simple. Therefore, if a monad represents the world, any well founded phenomenon does so, and if any monad has its own peculiarity this is also reflected in any well founded phenomenon. Beside this general truth, Leibniz is also explicit that he applies the PII to any piece of being, to leafs or to drops of water and any part of nature at all (Leibniz, 1989, p. 687).

Now the relation with principle of individuation goes as follows. In order to explain the meaning of identity as sameness and equivalence we have spoken about the objectifying intention. Such an intention works to distinguish a domain of objectivity as a unity and constitutes it as a specific entity. In this regard most things that we consider under the category of substantivity are not indeed monads perhaps even not well founded phenomena. Thus, we have a concept of identity which depends on our act of individuation. Besides the ontological identity, the ego may constitutes objects as identical to themselves in the course of everyday life as well as in scientific activity. To this level belong the notions of sameness and equivalence. However, in order to be able to perform the act of distinguishing and thus positing the equivalence or sameness, it is required that there be objective differences to serve ground as further distinctions. The point is that the metaphysical principle of identity of indiscernibles is itself recalled while explaining the possibility of having individuated identity. The principle of individuation says that the ego individuates a part of being on the bases of some properties and by means of his own acts. Nevertheless:

> The principle of individuation comes down to the principle of distinctness ... . If two individuals were perfectly alike—entirely indistinguishable in themselves—there wouldn't be any principle of individuation i.e. any basis for telling them apart. (Leibniz, 2008, p. 108)

Here Leibniz uses the term principle of distinctness instead of principle of identity of indiscernibles. As I said above the main significance of the latter is that of the former. From the PII, by contraposition, we have:

$$\forall x \forall y (x \neq_i y \rightarrow \exists F ((Fx \wedge \neg Fy) \vee (Fy \wedge \neg Fx))) \tag{8}$$

As I said above, since the antecedent is true for every pair of objective beings then the PII is indeed the principle of distinctness. The predicates here are those ontologically grounded. However there is no need to be so much concerned about it, since most of predicates (monadic or relational) are ontologically grounded, as the above passage itself says. However, we have a case of illusive phenomena and totally arbitrary attributes. Leibniz' method itself provides us with the means to depart to distinguish such inauthentic predicates from the genuine ones. But there is no need that we include a restriction in our formula; we should just be careful to use acceptable predicates in our language and we are already supposed to do that. But it is not true to think that epistemic predicates or relational ones should be ruled out, only those which are deliberately groundless or those which are proven not to be able to genuinely exist. [3] Even relational properties as far as grounded on the things themselves constitute genuine predicate. [4]

The connection with the principle of sufficient reason is also very important. If this principle would be true then in order to chose between two things there must be a reason, because no arbitrary action is admitted. Such a reason should refer to something in the choices themselves (as in the case of principle of individuation any preference in subjective view is grounded on objective difference). Then in order the principle of sufficient reason to be valid, every two things must be different. So the principle of identity of indiscernibles is the ontological counterpart of the principle of sufficient reason.

None of the above metaphysical considerations is relevant in the case of the indiscerniblity of identicals. Then again to consider II* and PII as two versions of the same thing is just to neglect the all philosophical significance of the PII.

Having carefully distinguished between the different meanings of the notion of

3. In this respect, we may disagree with Leibniz on some instances while being in agreement in principle. So, the debate on the genuineness of a specific property should not be included in a discussion on the PII itself. For example, Leibniz rejects that space can be well founded phenomenon. However, what he rejects to be a genuine existence is Newtonian account of absolute space, for it is homogeneous in itself and it violates the principle of distinctness. However, if we accept the conception of space that general theory of relativity introduces, the things would differ. This "space"may meet the Leibnizian criterion of discerniblity.

4. The case of relation is somewhat delicate since it ultimately requires the attribute shared by two or more substances. However Leibniz' latter works show that such attributes are possible and there is no contradiction in this respect with the principles of monadology (see (Mugnai, 1997)).

identity and having got an idea of what the PII is to say, we are now ready to go to formal treatments.

First of all, concerning the symbol “=Ï shall say that it is somehow pointless to reserve it only for ontological identity, because as we saw, the ontological identity is not indeed a relation. We normally use this symbol in the case of sameness and equivalence, and in this sense it can serve to form a predicate: $a = b$ means that a fulfills the objectifying intention that constitutes b. And also we have $\forall x \;\; x = x$, which means that any individual fulfills the objectifying intention which individuates it. So, in the rest of this writing I just use the symbol “=”without indexes, provided that the PII holds for any interpretation of =, and II* does not hold in general. These are to be shown in our semantic framework.

In the following I briefly introduce the dialogical semantics. Then I present rules concerning “=” and quantifying over predicates, provided that “=” may form a predicate.

3 A short introduction to dialogical semantics

I bring a short introduction of the dialogical semantics in the following.[5] Here I just describe the rules and do not discuss the philosophical advantages of the dialogical framework. For a good overview of the philosophical features of dialogical logic see (Rückert, 2001).

In dialogical semantics there are two parties, the proponent **P** and the opponent **O**. The proponent introduces a thesis and defends it against the attacks of the opponent. If there is a winning strategy for the proponent in respect to a statement, that statement is valid. The attacks and responses are to be performed according to two kinds of rules, particle rules and structural rules.

Particle Rules For any logical connective there is a particle rule which determines how to attack and defend a formula with a specific main connective. These rules are standard in the literature:

5. The following rules are standard within dialogical studies. However in the current manner of presentation I particularly benefited from the representations given in Rebuschi (2009) and Rahman (2005).

	Attack	Response
$A \vee B$	$?_{\vee}$	A, or B (The defender chooses)
$A \wedge B$	$?_{L}$, or $?_{R}$ (The attacker chooses)	A, or B (respectively)
$A \rightarrow B$	A	B
$\neg A$	A	(No possible respond)
$\forall x A$	$?_{\forall x/c}$ (The attacker chooses c)	$A[x/c]$
$\exists x A$	$?_{\exists x}$	$A[x/c]$ (The defender chooses c)

Structural Rules Structural rules determines the structure of the interaction which form a certain argumentation. We have:

(SR-0) Starting Rule: The Proponent begins by asserting a thesis.

(SR-1) Move: The players make their moves alternately. Each move, with the exception of the starting move, is an attack or a defense.

(SR-2) Winning Rule: Player **X** wins iff it is **Y**'s turn to play and **Y** cannot perform any move.

(SR-3) No Delaying Tactics Rule: Both players can only perform moves that change the situation.

(SR-4) Formal Rule: **P** cannot introduce any new atomic formula; new atomic formulas must be stated by **O** first. Atomic formulas can never be attacked.

(SR-5c) Classical Rule: In any move, each player may attack a complex formula uttered by the other player or defend him/herself against *any attack* (including those that have already been defended).

(SR-5i) Intuitionistic Rule: In any move, each player may attack a complex formula uttered by the other player or defend him/herself against *the last attack that has not yet been defended.*

Perhaps it would be good to give a simple example to show how this semantics works.[6] Let us examine the formula $p \rightarrow (q \rightarrow p)$.

6. In the following dialogues I just sketch out the core play which can be used to build a winning strategy. For the sake of simplicity I do not discuss all possible plays to show that in any case there is a winning strategy for **P** (or for **O** if I show the thesis is not valid); it is not difficult to show and I omit it in order to focus on the core of the argument. As regards the *repetition ranks* which concerns the rule SR-3, Clerbout (2014) has shown that it would be sufficient to assign the rank 1 to the opponent and 2 to the proponent, namely there would be a winning strategy for a formula if and only if there is a winning strategy for that formula while the proponent is allowed to attack

Remark on notation: The moves are brought in the order of utterance. The parenthesized information indicates the number of the move, whether it is attack (ⓐ) or response (ⓡ) and to which move it is attack or response. The long dash indicates that there is no further move; and the participant on whose side it appears has been lost.

	O	**P**	
		$p \to (q \to p)$	(0)
(1ⓐ0)	p	$q \to p$	(2ⓡ1)
(3ⓐ2)	q	p	(4ⓡ3)
	———		

Here in the move 4, **P** is able to respond the attack, because **O** has asserted p before; and since there is no other move possible for **O**, **P** wins and the formula is demonstrated to be valid.

4 A higher order logic, affirming Leibniz' law

Now I introduce new rules to extend our dialogical semantics in order it to be able to cover formulas like PII. Then we need quantification over predicates and also a rule for identity. An identity of the form $a = a$ is an elementary formula; and on the basis of any individual α we have in our language we can form a predicate $= \alpha$. Our structural rule for identity goes as follows:

(SR-id) Identity predication: Both parties are allowed to assert an elementary proposition in the form $\alpha = \alpha$ if they need it in order to respond or attack. Namely **P** *can* introduce such an atomic formula even if it has not been stated by **O** before.

The rational behind this rule is clear. As we accept that self-identity is generally true the parties are free to assert it in the case.

As for quantification we have two new particle rules intuitively analogous to the rules on quantification over individuals:

twice against a same move and the opponent is allowed to do so only once. Therefore, I do not specify the ranks in the following dialogues, and one can suppose that it is 1 for **O** and 2 for **P**.

	Attack	Response
$\forall FA$	$?_{\forall F/\Psi}$ (The attacker chooses Ψ)	$A[F/\Psi]$
$\exists FA$	$?_{\exists F}$	$A[F/\Psi]$ (The defender chooses Ψ)

Now it is easy to show that $\forall x \ \ x = x$ is valid:

	O	**P**	
		$\forall x \ \ x = x$	(0)
(1ⓐ0)	$?_{\forall x/a}$	$a = a$	(2ⓡ1)
	————		

Here **P** wins and the thesis is shown to be valid. **P** wins because for any attack that **O** makes, **P** can respond, since he is allowed to assert any atomic proposition of the form $\alpha = \alpha$.

Another formula which is proven to be valid is $\forall x \exists F Fx$. It says that any individual possesses at least one predicate. Let us verify it in our semantic framework.

	O	**P**	
		$\forall x \exists F Fx$	(0)
(1ⓐ0)	$?_{\forall x/a}$	$\exists F Fa$	(2ⓡ1)
(3ⓐ2)	$?_{\exists F}$	$a = a$	(4ⓡ3)
	————		

Here in the move 4, **P** chooses the predicate $= a$ to substitute F and since the result is a proposition in the form $\alpha = \alpha$ which is permitted to be asserted, **P** is able to answer any attack, then **O** loses and the formula is valid.

Now let us see the case for our main question, the formula expressing the PII:

$$\forall x \forall y (\forall F (Fx \leftrightarrow Fy) \rightarrow x = y) \quad (9)$$

O		P	
		$\forall x \forall y(\forall F(Fx \leftrightarrow Fy) \rightarrow x = y)$	(0)
(1ⓐ0)	$?_{\forall x/a}$	$\forall y(\forall F(Fa \leftrightarrow Fy) \rightarrow a = y)$	(2ⓡ1)
(3ⓐ2)	$?_{\forall y/b}$	$\forall F(Fa \leftrightarrow Fb) \rightarrow a = b$	(4ⓡ3)
(5ⓐ4)	$\forall F(Fa \leftrightarrow Fb)$	$?_{\forall F/=b}$	(6ⓡ5)
(7ⓡ6)	$a = b \leftrightarrow b = b$	$b = b$	(8ⓐ7)
(9ⓡ8)	$a = b$	$a = b$	(10ⓡ5)
	———		

As we see **P** wins and the PII is demonstrated to be valid. In the above dialogue the move 7 is a biconditional, so in the move 8 **P** by asserting one side can request **O** to assert the other side, and since **O** has no other move to perform she asserts the atomic proposition $a = b$ which makes **P** able to assert it and answer the attack posed by the move 5. In move 8, **P** is able to assert $b = b$ since it expresses a self-identity. So, for the PII there is a winning strategy and thus it is valid.

Now let us examine the formula II*:

$$\forall x \forall y(x = y \rightarrow \forall F(Fx \leftrightarrow Fy)) \tag{10}$$

O		P	
		$\forall x \forall y(x = y \rightarrow \forall F(Fx \leftrightarrow Fy))$	(0)
(1ⓐ0)	$?_{\forall x/a}$	$\forall y(a = y \rightarrow \forall F(Fa \leftrightarrow Fy))$	(2ⓡ1)
(3ⓐ2)	$?_{\forall y/b}$	$a = b \rightarrow \forall F(Fa \leftrightarrow Fb))$	(4ⓡ3)
(5ⓐ4)	$a = b$	$\forall F(Fa \leftrightarrow Fb)$	(6ⓡ5)
(7ⓐ6)	$?_{\forall F/\Phi}$	$\Phi a \leftrightarrow \Phi b$	(8ⓡ7)
(9ⓐ8)	Φa	———	

Here **O** in the move 7 chooses an arbitrary predicate, Φ, and demands **P** to replace it in the formula 6. Since **O** is allowed to assert any atomic formula, including Φa, she is able to attack the move 8. **P** is not able to respond, since Φb is an atomic formula and it has not been asserted by **O** before, and there is no other possibility for him to move, then **O** wins; namely the thesis shown not to be valid.

5 Analysis

As we expected our semantic framework admits the PII and rejects the II*. One may think that there can be added some rules to make II* also valid. Technically it may be true but the point is we employ dialogical semantics in order to explain the nature of the notions at work, and such rules would violate the all philosophical motivations behind dialogical semantics. Let us see what that rule should be. It would say that if a player **X** has asserted $a = b$ somewhere and also Fa in another occasion, the other player may ask **X** to assert Fb. But is this acceptable? Imagine that in a dialogue one says "water is H_2O", and in another occasion he or she says "it was in 1811 that it was discovered for the first time that water is H_2O". Now is it plausible that the other expect that he or she be able to assert "it was in 1811 that it was discovered for the first time that water is water¿ No, the two first assertions are true but this latter is not. More importantly the intention behind this latter has nothing to do with the intention behind the formers. So if someone says an identity relation $a = b$ and also one says a complex formula containing the individual a it is not reasonable to have such a rule to oblige one to assert that formula now about b.

Our structural rule for identity and the particle rules for quantifying over predicates are fairly intuitive. They validate the PII and they reject the II*. So our semantics can be admitted by the Leibnizian metaphysics and it is appropriate to be used in discussions concerning the metaphysics from Leibnizian point of view. And from the other side phenomenologically appropriateness of dialogical method and the intuitiveness of our new rules may support in its turn the rationale behind Leibnizian metaphysics and here specially the principle of identity of indiscernibles.

By means of the PII, Leibniz rejects both atomism and homogeneity in nature. Therefore for him being is an inhomogeneous continuity. Here we have ontological identity, which ultimately rests on monads and well founded phenomena, and also identity by individuation which depends on both ontological identity and diversity and the acts of the ego. In order to study such an ontology by logical means, we need a higher order logic with specific account of identity and the relation between identity and predicate. I hope that the dialogical logic just described would be able to contribute to such logical studies.

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